Appendix for
An Empirical Framework for Matching with Imperfect Competition.

Mons Chan[1], Kory Kroft[2], Elena Mattana[3], Ismael Mourifié[4]

Appendix A. Additional derivations and results.

A.1. **Optimal wage.** Under Assumption 2 (ii-a) the Karush-Kuhn-Tucker (KKT) necessary conditions for optimality of the firm's optimization problem are given by:[5]

(A-1) $\ell_{kj} + w_{kj}\frac{\partial \ell_{kj}}{\partial w_{kj}} - \lambda_j \frac{\partial \ell_{kj}}{\partial w_{kj}} F_k^j(\ell_{\cdot j}) \geq 0,$
(A-2) $w_{kj} \geq 0,$
(A-3) $w_{kj}\left[\ell_{kj} + w_{kj}\frac{\partial \ell_{kj}}{\partial w_{kj}} - \lambda_j \frac{\partial \ell_{kj}}{\partial w_{kj}} F_k^j(\ell_{\cdot j})\right] = 0,$
(A-4) $F^j(\ell_{\cdot j}) - Y_j \geq 0,$
(A-5) $\lambda_j \geq 0,$
(A-6) $\lambda_j\left[F^j(\ell_{\cdot j}) - Y_j\right] = 0$, for all $(k, j) \in (\mathcal{K} \times \mathcal{J})$.

Notice that given our ARUM and since u_{kj} is finite, $w_{kj} = 0$ implies that $\ell_{kj} = 0$. Under Assumptions 2 (i)-(ii-b), (A-4) is not violated if there exist some k such $\ell_{kj} > 0$ which means $w_{kj} > 0$ under Assumption 1. This means that each firm that is observed in this market pays a strictly positive wage to some types of worker. Let $\mathcal{C}^j \subseteq \mathcal{K}$ denote the set of worker types for whom firm j offers a strictly positive wage, $w_{kj} > 0$ which according our ARUM specification and Assumption 1 is equivalent to $s_{kj} > 0$. Then we have $\mathcal{C}^j \equiv \{k \in \mathcal{K} : s_{kj} > 0\}$. Then, (A-3) implies that (A-1) holds as an equality for all $k \in \mathcal{C}^j$ and thus $\ell_{kj} > 0$ for all $k \in \mathcal{C}^j$. We then have

$$w_{kj} = \lambda_j F_k^j(\ell_{\cdot j}) \frac{\mathcal{E}_{kj}}{1 + \mathcal{E}_{kj}}, \text{ for all } k \in \mathcal{C}^j \tag{A.1}$$

In this case, firm j optimally chooses to offer a wage equal to 0 when A-1 holds with strict inequality which corresponds to the case where the marginal cost for this type of worker exceeds the marginal product. Also, notice that all the RHS terms have to be positive to ensure that A-4 holds, which is compatible with the previous assumption used in the model.

A.2. **Recovering unobserved types.** The proposed identification strategy requires us to observe at least two time periods. We consider the following potential outcomes model:

$$Y_{it} = \sum_{j \in \mathcal{J}_0} [\ln w_{\mathbf{k}jt} + \eta_{ijt}] 1\{D_{it} = j\}, \quad t \in \{1, ..., T\} \tag{A.2}$$

[1]Department of Economics, Queen's University. 317 Dunning Hall 94 University Ave Kingston, ON, K7L 3N6, mons.chan@queensu.ca.
[2]Department of Economics, University of Toronto, & NBER. 150 St. George Street, Toronto ON M5S 3G7, Canada, kory.kroft@utoronto.ca.
[3]Department of Economics and Business Economics Aarhus University, Fuglesangs Allé 4 8210 Aarhus V Denmark, emattana@econ.au.dk.
[4]Department of Economics, Washington University in St. Louis & NBER, ismaelm@wustl.edu.
[5]Notice that in the case where the production functions are non-differentiable (for instance the Leontief Production function) sub-differential versions of KKT conditions are available and can be applied.

where Y_{it} denotes the observed log earnings of individual i at time t, and $1\{\cdot\}$ denotes the indicator function. $Y_{ijt} \equiv \ln w_{\mathbf{k}jt} + \eta_{ijt}$ denotes potential log earnings if individual i was externally assigned to work at firm j in period t. The potential outcomes are decomposed into two parts (i) $\ln w_{\mathbf{k}jt}$ is the log equilibrium wage, and (ii) η_{ijt} is measurement error or an i.i.d. worker-firm match effect realized after potential mobility across periods.

While in the main text we assumed that the worker's type k is observed by both firms and the econometrician, in general, we could allow k to consist of two subgroups of types, i.e. $k \equiv (\bar{k}, \tilde{k})$, where $\bar{k}$ is defined based on the underlying vector of characteristics $\overline{X}$ that are observed both by the econometrician and firms while $\tilde{k}$ is defined based on the set of characteristics $\widetilde{X}$ that are observable only to firms but not to the econometrician.

Let m_{it} denote the mobility variable, more precisely $m_{it} = 1$ iff $D_{it} \neq D_{it+1}$, i.e. $m_{it} = 1\{D_{it} \neq D_{it+1}\}$. Using shorthand notation $\bar{\mathbf{k}}^{\mathbf{t+1}} = (\bar{\mathbf{k}}_{\mathbf{t}}, \bar{\mathbf{k}}_{\mathbf{t+1}})$, consider the following assumption:

Assumption 4 (Time invariance, Mobility, and Serial Dependence)**.** *We impose the following restrictions.*

(i) Time invariance of unobserved types: $\tilde{\mathbf{k}}_{\mathbf{t}} = \tilde{\mathbf{k}}$ *for* $t \in \{1, ..., T\}$.
(ii) Classical errors: $(\eta_{ijt}, \eta_{ilt+1}) \perp (D_{it}, D_{it+1}) | \tilde{\mathbf{k}}, \bar{\mathbf{k}}_{\mathbf{t}}, \bar{\mathbf{k}}_{\mathbf{t+1}}$
(iii) No serial dependence in the errors: $\eta_{ijt} \perp \eta_{ilt+1} | \tilde{\mathbf{k}}, \bar{\mathbf{k}}_{\mathbf{t}}, \bar{\mathbf{k}}_{\mathbf{t+1}}$ *and* $\eta_{ijt} \perp \bar{\mathbf{k}}_{\mathbf{t+1}} | \tilde{\mathbf{k}}, \bar{\mathbf{k}}_{\mathbf{t}}$

Assumption 4(i) requires the unobserved types to be time invariant. In the same spirit as Burdett and Mortensen (1998) and Hagedorn et al. (2017), Assumption 4(ii) requires the errors to not be correlated with sorting and mobility decisions. The intuition is that these errors are realized after the matches between workers and firms have been formed. Assumption 4(iii) requires the measurement errors associated to a specific mover to not be serially dependent.

Under Assumption 4 we can show that

$$\begin{aligned} &\mathbb{P}(Y_{it} \leq y_t, Y_{i,t+1} \leq y_{t+1} | D_{it} = j, D_{it+1} = l, m_{it} = 1, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}) \\ &\qquad = \sum_{\tilde{k}} \mathbb{P}_{\tilde{k}j}(y_t|\bar{k}_t) \mathbb{P}^m_{\tilde{k}l}(y_{t+1}|\bar{k}^{t+1}) \mathbb{P}(\tilde{\mathbf{k}} = \tilde{k} | D_{it} = j, D_{it+1} = l, m_{it} = 1, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}) \end{aligned} \quad \text{(A.3)}$$

where

$$\mathbb{P}_{\tilde{k}j}(y_t|\bar{k}_t) \equiv \mathbb{P}(Y_{it} \leq y_t | D_{it} = j, \tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}} = \bar{k}_t), \quad \text{(A.4)}$$

$$\mathbb{P}^m_{\tilde{k}l}(y_{t+1}|\bar{k}^{t+1}) \equiv \mathbb{P}(Y_{i,t+1} \leq y_{t+1} | D_{it+1} = l, m_{it} = 1, \tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}). \quad \text{(A.5)}$$

Whenever the above decomposition holds and the following three requirements hold: (i) Any two firms j and l belong to a connecting cycle as formally defined in Bonhomme et al. (2019), Definition 1, (ii) there exists some asymmetry in the worker type composition between different firms, i.e, Bonhomme et al. (2019), Assumption 3(i), and (iii) the matrix defined by the joint log earning distribution $\mathbb{P}(Y_{it} \leq y_t, Y_{i,t+1} \leq y_{t+1} | D_{it} = j, D_{it+1} = l, m_{it} = 1, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1})$ for different values of (y_t, y_{t+1}) respects a certain rank condition, i.e, Bonhomme et al. (2019), Assumption 3(ii). Then Theorem 1 of Bonhomme et al. (2019) applies and the following quantities are point identified: $\mathbb{P}_{\tilde{k}j}(y_t|\bar{k}_t)$, $\mathbb{P}^m_{\tilde{k}l}(y_{t+1}|\bar{k}_{t+1})$, and $\mathbb{P}_{jt}(\tilde{k}|\bar{k}_t) \equiv \mathbb{P}(\tilde{\mathbf{k}} = \tilde{k} | D_{it} = j, \bar{\mathbf{k}}_{\mathbf{t}} = \bar{k}_t)$.

These distributions can be parametrically estimated using the EM algorithm entertained in Bonhomme et al. (2019). Using this identification result, it is possible to recover equilibrium wages and shares that were initially unobserved to the econometrician. More precisely, we have the following result:

Proposition 3 (Identification of equilibrium wages and shares)**.** *Consider Assumption 4 holds, and the cdf of classical errors $F_{\eta_{ijt}|\mathbf{k_t}=k_t}(.)$, and $F_{\eta_{ilt+1}|\mathbf{k^{t+1}}=k^{t+1}}(.)$ are known and strictly increasing on $\mathbb{R}$. If the following quantities are point identified $\mathbb{P}_{\tilde{k}j}(y_t|\bar{k}_t)$, $\mathbb{P}^m_{\tilde{k}l}(y_{t+1}|\bar{k}_{t+1})$, $\mathbb{P}_{jt}(\tilde{k}|\bar{k}_t)$; then we have the following identification result:*

$$w_{kjt} = \exp\left\{y_t - F^{-1}_{\eta_{ijt}|\mathbf{k_t}=k_t}\left(\mathbb{P}_{\tilde{k}j}(y_t|\bar{k}_t)\right)\right\}, \tag{A.6}$$

$$w_{klt+1} = \exp\left\{y_{t+1} - F^{-1}_{\eta_{ilt+1}|\mathbf{k^{t+1}}=k^{t+1}}\left(\mathbb{P}^m_{\tilde{k}l}(y_{t+1}|\bar{k}_{t+1})\right)\right\}, \tag{A.7}$$

$$s_{kjt} = \mathbb{P}_{jt}(\tilde{k}|\bar{k}_t)\frac{s_{\bar{k}jt}}{\sum_{\mathcal{J}_0}\mathbb{P}_{jt}(\tilde{k}|\bar{k}_t)s_{\bar{k}jt}}. \tag{A.8}$$

where $s_{kjt} = \mathbb{P}(D_{it} = j|\mathbf{k_t} = k_t)$ and $s_{\bar{k}jt} = \mathbb{P}(D_{it} = j|\bar{\mathbf{k}}_\mathbf{t} = \bar{k}_t)$

Proof of Proposition 3.

$$\begin{aligned}
&\mathbb{P}(Y_{it} \leq y_t, Y_{i,t+1} \leq y_{t+1}|D_{it} = j, D_{it+1} = l, m_{it} = 1, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}) \\
&\quad= \sum_{\tilde{k}} \mathbb{P}(Y_{it} \leq y_t, Y_{i,t+1} \leq y_{t+1}|D_{it} = j, D_{it+1} = l, \tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1})\times \\
&\quad\quad \overbrace{\mathbb{P}(\tilde{\mathbf{k}} = \tilde{k}|D_{it} = j, D_{it+1} = l, \bar{\mathbf{k}}_\mathbf{t} = \bar{k}_t, \bar{\mathbf{k}}_\mathbf{t+1} = \bar{k}_{t+1})}^{P(\tilde{k}|j,l,\bar{k}^{t+1})} \\
&= \sum_{\tilde{k}} \mathbb{P}(\ln w_{\mathbf{k}jt}+\eta_{ijt} \leq y_t, \ln w_{\mathbf{k}j,t+1}+\eta_{ilt+1} \leq y_{t+1}|D_{it} = j, D_{it+1} = l, \tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}) \times P(\tilde{k}|j,l,\bar{k}^{t+1}) \\
&\quad= \sum_{\tilde{k}} \mathbb{P}\left(\ln w_{\mathbf{k}jt} + \eta_{ijt} \leq y_t, \ln w_{\mathbf{k}j,t+1} + \eta_{ilt+1} \leq y_{t+1}|\tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}\right) \times P(\tilde{k}|j,l,\bar{k}^{t+1}) \\
&= \sum_{\tilde{k}} \mathbb{P}\left(\ln w_{\mathbf{k}jt} + \eta_{ijt} \leq y_t|\tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}\right)\times\mathbb{P}\left(\ln w_{\mathbf{k}j,t+1} + \eta_{ilt+1} \leq y_{t+1}|\tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}\right)\times P(\tilde{k}|j,l,\bar{k}^{t+1}) \\
&\quad= \sum_{\tilde{k}} \mathbb{P}\left(\ln w_{\mathbf{k}jt} + \eta_{ijt} \leq y_t, \ln w_{\mathbf{k}j,t+1} + \eta_{ilt+1} \leq y_{t+1}|\tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}\right) \times P(\tilde{k}|j,l,\bar{k}^{t+1}) \\
&= \sum_{\tilde{k}} \mathbb{P}\left(Y_{it} \leq y_t|D_{it} = j, \tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}_\mathbf{t} = \bar{k}_t\right)\times\mathbb{P}\left(Y_{i,t+1} \leq y_{t+1}|D_{it+1} = l, m_{it} = 1, \tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}\right)\times P(\tilde{k}|j,l,\bar{k}^{t+1})
\end{aligned}$$

Now, we have

$$\begin{aligned}
&\mathbb{P}_{\tilde{k}j}(y_t|\bar{k}_t) \equiv \mathbb{P}(Y_{it} \leq y_t|D_{it} = j, \tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}_\mathbf{t} = \bar{k}_t) \\
&= \mathbb{P}(\ln w_{\mathbf{k}jt}+\eta_{ijt} \leq y_t|D_{it} = j, \tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}_\mathbf{t} = \bar{k}_t) = \mathbb{P}(\ln w_{\mathbf{k}jt}+\eta_{ijt} \leq y_t|\tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}_\mathbf{t} = \bar{k}_t) = \mathbb{P}(\eta_{ijt} \leq y_t - \ln w_{\mathbf{k}jt}|\tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}_\mathbf{t} = \bar{k}_t) \\
&\qquad = F_{\eta_{ijt}|\bar{\mathbf{k}}_\mathbf{t}=\bar{k}_t}(y_t - \ln w_{\mathbf{k}jt})
\end{aligned}$$

We can then easily recover the first result by inverting the last equation and obtain: $w_{kjt} = \exp\left\{y_t - F^{-1}_{\eta_{ijt}|\bar{\mathbf{k}}_\mathbf{t}=\bar{k}_t}\left(\mathbb{P}_{\tilde{k}j}(y_t|\bar{k}_t)\right)\right\}$. The second equality of the proposition could be derived analogously. For the last equality we have:

$$\mathbb{P}(D_{it}=j|\tilde{\mathbf{k}}=\tilde{k},\bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)=\frac{\mathbb{P}(\tilde{\mathbf{k}}=\tilde{k}|D_{it}=j,\bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)\times\mathbb{P}(D_{it}=j|\tilde{\mathbf{k}}=\tilde{k},\bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)}{\mathbb{P}(\tilde{\mathbf{k}}=\tilde{k}|\bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)}$$
$$=\frac{\mathbb{P}(\tilde{\mathbf{k}}=\tilde{k}|D_{it}=j,\bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)\times\mathbb{P}(D_{it}=j|\tilde{\mathbf{k}}=\tilde{k},\bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)}{\sum_j\mathbb{P}(\tilde{\mathbf{k}}=\tilde{k}|D_{it}=j,\bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)\times\mathbb{P}(D_{it}=j|\tilde{\mathbf{k}}=\tilde{k},\bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)}$$

□

Parametric estimation and EM algorithm. For practical purposes, we impose a normality distribution for the classical errors, then $\ln w_{kjt}+\eta_{ijt}|\mathbf{k^t}=k^t\sim N\left(\ln w_{kjt},\varrho_{kjt}\right)$ and $\ln w_{klt}+\eta_{ilt+1}|\mathbf{k^{t+1}}=k^{t+1}\sim N\left(\ln w_{klt+1},\varrho_{klt+1}\right)$. Let $\tilde{K}$ denote the number of unobserved types, $C_{\bar{k}^t}$ be a set of firms that have been hiring workers of observable types $\bar{k}^t$ over the two periods t and $t+1$ and belonging to a connecting cycle as defined in Bonhomme et al. (2019). $N^m_{\bar{k}^t}$ denotes the number of movers with observable types $\bar{k}^t$. First, we consider the following log-likelihood function for job movers:

$$\sum_{i=1}^{N^m_{\bar{k}^t}}\sum_{j\in C_{\bar{k}^t}}\sum_{l\in C_{\bar{k}^t}}\ln\left(\sum_{\tilde{k}=1}^{\tilde{K}}p_{\tilde{k}jl}\frac{1}{\sqrt{4\pi^2\varrho_{(\tilde{k},\bar{k}_t)jt}\varrho_{(\tilde{k},\bar{k}_t)lt+1}}}e^{-\frac{\left(y_{it}-\ln w_{(\tilde{k},\bar{k}_t)jt}\right)^2}{2\varrho^2_{(\tilde{k},\bar{k}_t)jt}}-\frac{\left(y_{it+1}-\ln w_{(\tilde{k},\bar{k}_t)lt+1}\right)^2}{2\varrho^2_{(\tilde{k},\bar{k}_t)lt+1}}}\right) \tag{A.9}$$

where $\hat{w}_{(\tilde{k},\bar{k}_t)jt}$, $\hat{w}_{(\tilde{k},\bar{k}_t)lt+1}$, $\hat{\varrho}_{(\tilde{k},\bar{k}_t)jt}$, $\hat{\varrho}_{(\tilde{k},\bar{k}_t)lt+1}$, and $\hat{p}_{\tilde{k}jl}$ for $\tilde{k}=1,...,\tilde{K}$ are estimated by maximizing (A.10) using the EM algorithm.

Second, we consider the log-likelihood of the for all workers at the period t:

$$\sum_{i=1}^{N_{\bar{k}^t}}\sum_{j\in C_{\bar{k}_t}}\ln\left(\sum_{\tilde{k}=1}^{\tilde{K}}q_{\tilde{k}jt}\frac{1}{\sqrt{4\pi^2\hat{\varrho}_{(\tilde{k},\bar{k}_t)jt}}}e^{-\frac{\left(y_{it}-\ln\hat{w}_{(\tilde{k},\bar{k}_t)jt}\right)^2}{2\hat{\varrho}^2_{(\tilde{k},\bar{k}_t)jt}}}\right) \tag{A.10}$$

where $N_{\bar{k}_t}$ denotes the number of workers with observable types $\bar{k}_t$, and $q_{\tilde{k}jt}\equiv\mathbb{P}_{jt}(\tilde{k}|\bar{k}_t)$. Again we estimate $\hat{q}_{\tilde{k}jt}$ by maximizing eq (A.10) using the EM algorithm. Then we use eq (A.8) to recover $\hat{s}_{kjt}$.

Given employment shares s_{kjt} for each firm and worker type, we can then obtain the total quantity of each worker type in the population, $m_{kt}=\sum_j\ell_{kjt}$, as the (year-by-year) solution to an overdetermined system of linear equations: $S_tm_t=\mu_t$. Here S_t is the known $J\times K$ matrix of worker type shares s_{kjt} at each firm in period t, μ_t is the known $J\times 1$ vector of total employment $\mu_{jt}=\sum_{k\in\mathcal{C}^j_t}\ell_{kjt}$ at each firm, and m_t is the unknown $K\times 1$ vector of individuals m_{kt} of each type k. If both S_t and the associated augmented matrix have rank equal to K, then there will be a unique solution which provides m_{kt} for each period t[6]. We can then obtain $\ell_{kjt}=s_{kjt}m_{kt}$ for each firm, type and year.

Given that we have recovered the equilibrium wages and shares, and number of matches, these objects can then be used to recover the model parameters.

A.3. **Identifying the Labor Supply Parameters.**

[6]This is the Rouché-Capelli theorem.

A.3.1. *Estimating the Supply Equation.* The baseline labor supply equation from the model is

$$\ln \frac{s_{kjt}}{s_{k0t}} = \overline{u}_k + \beta_{1k} \ln \frac{w_{kjt}}{w_{k0t}} + \sum_{g=1}^{G} \tilde{\sigma}_{kg} \ln s_{kj|gt} \mathbb{1}_{j|g} + \ln u_{kjt} \tag{A.11}$$

where $\tilde{\sigma}_{kg} \equiv (1 - 1/\sigma_{kg})$. Define $\mathbb{1}_{j|g} = 1$ if $j \in g$ and 0 else.

The identification challenge is that both the wage and inside share are potentially correlated with the unobserved amenities and thus endogenous. To address this challenge, we propose an instrumental variables (IV) strategy which leverages exogenous variation in firm productivity. Before discussing this IV strategy, we review candidate instruments which we considered.

One source of instruments relies on strategic interactions between firms in wage setting. In the presence of strategic interactions, the number and characteristics of other firms in a given labor market can be used as instruments. These so-called "BLP instruments" are very common in the industrial organization literature in the context of the product market where the characteristics and number of competing products are used as instruments for prices (see Berry et al. (1995) (BLP) for the canonical example). In a labor market context, possible BLP instruments might include the number of firms, average size, or average value-added per worker of other firms in the labor market. Azar et al. (2022a) use the number of vacancies and log employment of competing firms as instruments for advertised wages on a job posting website. In results not reported, we consider the available BLP instruments in our data, such as the number of firms in the same market, and found that they were not sufficiently strong. Thus, we do not emphasize BLP instruments in our setting.

A second source of wage instruments exploits "uniform wage setting" whereby firms set wages similarly across local labor markets (Hazell et al., 2022). As suggested by Azar et al. (2022a), this implies that the wage a firm pays in a given market may be driven by the labor market conditions that same firm faces in other markets. We thus considered Hausman instruments for w_{kjg} in market g using the average predicted wage across all markets that firm operates in other than g[7]. In results not reported, we implemented this approach, following Azar et al. (2022a), but generally found that these instruments were too weak in our setting.

Finally, we considered a shift-share IV approach following Hummels et al. (2014) and Garin and Silvério (2023) to estimate labor supply. To construct this instrument, we rely on firm-product-country level yearly foreign trade data from Statistics Denmark register UHDI and bilateral trade flows from the BACI dataset. We find that our labor supply parameters are comparable to our main estimates reported in Table D.5. We do not emphasize these estimates as much in the paper since we are only able to construct the instrument for the small share of the firms in our sample who export. These results are available upon request.

For any of those approaches, let's present how the parameters can be consistently estimated.

A.3.2. *Estimating the Supply Equation in Changes.* We can rewrite the supply equation in changes as

$$\Delta_{e,e'} \ln \frac{s_{kj|gt}}{s_{k0t}} = \beta_{0k} + \beta_{1k} \Delta_{e,e'} \ln \frac{w_{kj|gt}}{w_{k0t}} + \sum_{g=1}^{G} \tilde{\sigma}_{kgt} \Delta_{e,e'} \ln s_{kj|gt} \mathbb{1}_{j|g} + \Delta_{e,e'} \ln u_{kj|gt} \tag{A.12}$$

where $\Delta_{e,e'} x_t \equiv x_{t+e} - x_{t-e'}$.

[7]We also exclude markets in the same municipality or industry as g.

For ease of notation, we will fix a labor type k (dropping the notation) and pool observations across firms and years (and markets), replacing indices (j,t) with a single index $n \in 1,...,N$ representing total number of observations for labor type k. We define $\tilde{s}_n = \ln \frac{s_{jt}}{s_{0t}}$, $\tilde{w}_n = \ln \frac{w_{jt}}{w_{0t}}$, $\tilde{i}_{ng} = \ln s_{j|gt} \mathbb{1}_{j|g}$, and $\tilde{u}_n = \ln u_{jt}$. We can write this equation in matrix notation as

$$\underset{N\times 1}{\mathbf{S^\Delta}} = \underset{N\times 1}{\mathbf{X_0}}\beta_0 + \underset{N\times(G+1)}{\mathbf{X_1^\Delta}}\underset{(G+1)\times 1}{\boldsymbol{\beta}} + \underset{N\times 1}{\mathbf{U^\Delta}} \tag{A.13}$$

where $\mathbf{X_0}$ is a column vector of 1's,

$$\mathbf{S^\Delta} = \begin{pmatrix} \Delta_{e,e'}\tilde{s}_1 \\ \Delta_{e,e'}\tilde{s}_2 \\ \vdots \\ \Delta_{e,e'}\tilde{s}_N \end{pmatrix}, \quad \mathbf{X_1^\Delta} = \begin{pmatrix} \Delta_{e,e'}\tilde{w}_1 & \Delta_{e,e'}\tilde{i}_{11} & \cdots & \Delta_{e,e'}\tilde{i}_{1G} \\ \Delta_{e,e'}\tilde{w}_2 & \Delta_{e,e'}\tilde{i}_{21} & \cdots & \Delta_{e,e'}\tilde{i}_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta_{e,e'}\tilde{w}_N & \Delta_{e,e'}\tilde{i}_{N1} & \cdots & \Delta_{e,e'}\tilde{i}_{NG} \end{pmatrix}, \quad \mathbf{U^\Delta} = \begin{pmatrix} \Delta_{e,e'}\tilde{u}_1 \\ \Delta_{e,e'}\tilde{u}_2 \\ \vdots \\ \Delta_{e,e'}\tilde{u}_N \end{pmatrix}$$

Define $(\mathbf{W}^\Delta)^T = (\Delta_{e,e'}\tilde{w}_1, ..., \Delta_{e,e'}\tilde{w}_N)$ and $(\mathbf{I_g}^\Delta)^T = (\Delta_{e,e'}\tilde{i}_1 g, ..., \Delta_{e,e'}\tilde{i}_N g)$. Suppose we now want to use variable Δr_n to instrument for $\Delta_{e,e'}\tilde{w}_n$, and variable Δf_{ng} to instrument for $\Delta_{e,e'}\tilde{i}_{ng}$. Here, Δr_n is the one-period change in (log) firm revenues and Δf_{ng} is the one-period change in the (log) inside share in market g, where as above n indexes across j and t. Define the matrix of instruments $\mathbf{Z^\Delta}$ as

$$\mathbf{Z^\Delta} = \begin{pmatrix} \mathbf{R^\Delta} & \mathbf{F_1^\Delta} & \cdots & \mathbf{F_G^\Delta} \end{pmatrix} = \begin{pmatrix} \Delta r_1 & \Delta f_{11} & \cdots & \Delta f_{1G} \\ \Delta r_2 & \Delta f_{21} & \cdots & \Delta f_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta r_N & \Delta f_{N1} & \cdots & \Delta f_{NG} \end{pmatrix}$$

Given the intercept term, as above, we can write the instrumental variable estimator for $\boldsymbol{\beta}$ with the equation in changes as

$$\widehat{\boldsymbol{\beta}}^\Delta = \text{Cov}(\mathbf{Z^\Delta}, \mathbf{X_1^\Delta})^{-1}\text{Cov}(\mathbf{Z^\Delta}, \mathbf{S^\Delta}) \tag{A.14}$$

$$= \begin{pmatrix} \mathbf{C^\Delta_{RW}} & \mathbf{C^\Delta_{RI}} \\ \mathbf{C^\Delta_{FW}} & \mathbf{C^\Delta_{FI}} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{C^\Delta_{RS}} \\ \mathbf{C^\Delta_{FS}} \end{pmatrix} \tag{A.15}$$

where

$$\underset{1\times 1}{\mathbf{C^\Delta_{RW}}} = \text{Cov}(\mathbf{R^\Delta}, \mathbf{W^\Delta}), \quad \underset{1\times G}{\mathbf{C^\Delta_{RI}}} = \Big(\text{Cov}(\mathbf{R^\Delta}, \mathbf{I_1^\Delta}) \cdots \text{Cov}(\mathbf{R^\Delta}, \mathbf{I_G^\Delta})\Big) \tag{A.16}$$

and

$$\underset{G\times 1}{\mathbf{C^\Delta_{FW}}} = \begin{pmatrix} \text{Cov}(\mathbf{F_1^\Delta}, \mathbf{W^\Delta}) \\ \vdots \\ \text{Cov}(\mathbf{F_G^\Delta}, \mathbf{W^\Delta}) \end{pmatrix}, \quad \underset{G\times G}{\mathbf{C^\Delta_{FI}}} = \begin{pmatrix} \text{Cov}(\mathbf{F_1^\Delta}, \mathbf{I_1^\Delta}) \cdots \text{Cov}(\mathbf{F_1^\Delta}, \mathbf{I_G^\Delta}) \\ \vdots \ddots \vdots \\ \text{Cov}(\mathbf{F_G^\Delta}, \mathbf{I_1^\Delta}) \cdots \text{Cov}(\mathbf{F_G^\Delta}, \mathbf{I_G^\Delta}) \end{pmatrix} \tag{A.17}$$

and finally

$$\underset{1\times 1}{\mathbf{C^\Delta_{RS}}} = \text{Cov}(\mathbf{R^\Delta}, \mathbf{S^\Delta}), \quad \underset{G\times 1}{(\mathbf{C^\Delta_{FS}})^T} = \Big(\text{Cov}(\mathbf{F_1^\Delta}, \mathbf{S^\Delta}) \cdots \text{Cov}(\mathbf{F_G^\Delta}, \mathbf{S^\Delta})\Big) \tag{A.18}$$

What comes next requires a few assumptions:

Assumption 5. *The instruments are predetermined. i.e.:* $C^\Delta_{RU} \equiv Cov(\mathbf{R^\Delta}, \mathbf{U^\Delta}) = 0$ *and* $C^\Delta_{FU} \equiv Cov(\mathbf{F^\Delta}, \mathbf{U^\Delta}) = \mathbf{0}$.

Assumption 6. *The instruments are valid and correlated with the endogenous regressors. i.e.: The* $G \times G$ *matrix* $\mathbb{E}[(\mathbf{Z^\Delta})'\mathbf{X_1^\Delta}]$ *is full column rank.*

These two assumptions are similar to assumptions made in Lamadon et al. (2022) and Kroft et al. (2023), who also estimate labor supply systems in changes. Specifically, assumptions 5 and 6 together encompass assumption 3 in Kroft et al. (2023). Assumptions 5 and 6 are satisfied for each instrument z_{jkt} if (briefly using full notation) $\exists\, e\,,\, e' > 0$ such that $\mathrm{Cov}(\tilde{\gamma}_{kjt+e}-\tilde{\gamma}_{kjt-e'}, \Delta z_{jkt}) \neq 0$ and $\mathrm{Cov}(\ln u_{kjt+e}-\ln u_{kjt-e'}, \Delta z_{jkt}) = 0$. The first is satisfied if the firm productivity process is sufficiently persistent (i.e.: δ is sufficiently close to 1 under the AR(1) assumptions in section 5.2). The second is satisfied if the amenity process is sufficiently transitory. Lamadon et al. (2022) and Kroft et al. (2023) argue that unobserved firm-specific job amenity shocks are well approximated by an MA(1) process. Under this specification, $e \geq 2$ and $e' \geq 3$ satisfy the exclusion restrictions.

Given these assumptions, the estimator becomes

$$\widehat{\boldsymbol{\beta}}^{\Delta} = \begin{pmatrix} \widehat{\beta_1}^{\Delta} \\ \widehat{\tilde{\sigma}}^{\Delta} \end{pmatrix} = \begin{pmatrix} \frac{1}{\overline{C}^{\Delta}}\left(\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RS}} - \mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RI}}(\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}})^{-1}\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FS}}\right) \\ \frac{1}{\overline{C}^{\Delta}}\left((\overline{\mathbf{C}}^{\boldsymbol{\Delta}}(\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}})^{-\mathbf{1}} + (\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}})^{-\mathbf{1}}\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FW}}\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RI}}(\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}})^{-1})\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FS}} - (\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}})^{-1}\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FW}}\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RS}}\right) \end{pmatrix} \tag{A.19}$$

where $\overline{C}^{\Delta} \equiv \mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RW}} - \mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RI}}(\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}})^{-\mathbf{1}}\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FW}}$ is a non-zero scalar, since assumption 6 implies that $\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RW}}$ is non-zero and $\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}}$ is invertible. We can then state the following result:

Proposition 4. *Under Assumptions 5 and 6, $\widehat{\beta}^{\Delta}$ recovers β.*

Proof. By equation A.13 we have:

$$\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RS}} = \mathrm{Cov}(\mathbf{R}^{\boldsymbol{\Delta}}, \mathbf{S}^{\boldsymbol{\Delta}}) = \mathrm{Cov}(\mathbf{R}^{\boldsymbol{\Delta}}, \mathbf{W}^{\boldsymbol{\Delta}}\beta_1^{\Delta} + \mathbf{I}\tilde{\boldsymbol{\sigma}}^{\boldsymbol{\Delta}} + \mathbf{U}^{\boldsymbol{\Delta}})$$

and

$$\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FS}} = \mathrm{Cov}(\mathbf{F}^{\boldsymbol{\Delta}}, \mathbf{S}^{\boldsymbol{\Delta}}) = \mathrm{Cov}(\mathbf{F}^{\boldsymbol{\Delta}}, \mathbf{W}^{\boldsymbol{\Delta}}\beta_1^{\Delta} + \mathbf{I}\tilde{\boldsymbol{\sigma}}^{\boldsymbol{\Delta}} + \mathbf{U}^{\boldsymbol{\Delta}})$$

By equation A.19 and assumption 6, the estimator $\widehat{\beta}_1^{\Delta}$ is thus

$$\begin{aligned}
\widehat{\beta}_1^{\Delta} &= \frac{1}{\overline{C}^{\Delta}}\left(\beta^{\boldsymbol{\Delta}}_{\mathbf{1}}\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RW}} + \mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RI}}\tilde{\boldsymbol{\sigma}}^{\boldsymbol{\Delta}} + \mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RU}} - \mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RI}}(\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}})^{-1}(\beta^{\boldsymbol{\Delta}}_{\mathbf{1}}\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FW}} + \mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}}\tilde{\boldsymbol{\sigma}}^{\boldsymbol{\Delta}} + \mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FU}})\right) \\
&= \frac{1}{\overline{C}^{\Delta}}\left(\beta^{\boldsymbol{\Delta}}_{\mathbf{1}}(\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RW}} - \mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RI}}(\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}})^{-\mathbf{1}}\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FW}}) + (\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RI}} - \mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RI}}(\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}})^{-\mathbf{1}}\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}})\tilde{\boldsymbol{\sigma}}^{\boldsymbol{\Delta}}\right) \\
&= \beta_1^{\Delta}\frac{\overline{C}^{\Delta}}{\overline{C}^{\Delta}} + \frac{1}{\overline{C}^{\Delta}}\mathbf{0}\tilde{\boldsymbol{\sigma}}^{\boldsymbol{\Delta}} \\
&= \beta_1^{\Delta}
\end{aligned}$$

where the second equation is due to assumption 5, and the third equation is due to the definition of $\overline{C}^{\Delta}$. Similarly, by assumption 6, for $\tilde{\boldsymbol{\sigma}}^{\boldsymbol{\Delta}}$ we have

$$\begin{aligned}
\widehat{\tilde{\boldsymbol{\sigma}}^{\boldsymbol{\Delta}}} &= \frac{1}{\overline{C}^{\Delta}}\Big((\overline{\mathbf{C}}^{\boldsymbol{\Delta}}(\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}})^{-1} + (\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}})^{-1}\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FW}}\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RI}}(\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}})^{-1})(\beta^{\boldsymbol{\Delta}}_{\mathbf{1}}\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FW}} + \mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}}\tilde{\boldsymbol{\sigma}}^{\boldsymbol{\Delta}} + \mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FU}}) + \\
&\qquad - (\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}})^{-\mathbf{1}}\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FW}}(\beta^{\boldsymbol{\Delta}}_{\mathbf{1}}\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RW}} + \mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RI}}\tilde{\boldsymbol{\sigma}}^{\boldsymbol{\Delta}} + \mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RU}})\Big) \\
&= \frac{\overline{C}^{\Delta}}{\overline{C}^{\Delta}}\tilde{\boldsymbol{\sigma}}^{\boldsymbol{\Delta}} + \frac{1}{\overline{C}^{\Delta}}\beta_1^{\Delta}(\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}})^{-\mathbf{1}}\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FW}}(\overline{C}^{\Delta} + \mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RI}}(\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}})^{-\mathbf{1}}\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FW}} - \mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{RW}}) \\
&= \tilde{\boldsymbol{\sigma}}^{\boldsymbol{\Delta}} + \frac{1}{\overline{C}^{\Delta}}\beta_1^{\Delta}(\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FI}})^{-\mathbf{1}}\mathbf{C}^{\boldsymbol{\Delta}}_{\mathbf{FW}}(\overline{C}^{\Delta} - \overline{C}^{\Delta}) \\
&= \tilde{\boldsymbol{\sigma}}^{\boldsymbol{\Delta}}
\end{aligned}$$

where again the second equality is due to assumption 5 and the third equality is due to the definition of $\overline{C}^{\Delta}$. □

A.4. **Multi-Equation GMM Approach to Estimating Production Parameters.** Estimating equation 5.6 is not straight forward. We cannot use an equation-by-equation approach as we do for the labor supply equation due to the presence of common parameters across equations. While there are only $K+1$ parameters to estimate ($\rho_k \,\forall\, k$ and δ), there are $K*(K-1)/2$ equations which could be used to estimate the parameters, with no obvious guidance on which to use. Since not all firms employ every labor type, any subset of equations will somewhat arbitrarily ignore the contribution of some firms. If all firms employed some base type of labor, all the labor ratio equations could be cast in terms of that type. However this is not the case, so an alternative is to use all $K*(K-1)/2$ equations in a multi-equation GMM estimator. Another possible approach would be to treat the multi-equation GMM system non-linearly and estimate the $K+1$ parameters directly. This would require $K+1$ instruments, for which the obvious choices are lagged labor and wages for each labor type. However, due to the size of the problem this may be intractable.

The approach we take is to treat the system as a set of linear equations with cross-equation parameter restrictions, estimating the compound parameters (such as $\delta(\rho_k-1)$) and then calculating the structural parameters post-estimation. This has the advantage of being much faster, and also allows specification testing of the model assumptions (since we can test if our estimates of $\delta(\rho_k-1)$ equal the product of our estimates of δ and (ρ_k-1)). Functionally, we estimate $K*(K-1)/2$ equations, where each equation (for all a,b in the set of labor types) takes the following form:

$$\begin{aligned} d_{kjt}d_{hjt}\log\frac{\tilde{w}_{ajt}}{\tilde{w}_{bjt}} &= \sum_{k}\mathbb{1}_{k=a}d_{kjt}\left[\beta_k^1\log\ell_{kjt}-\beta_k^2\log\mu_{kjt-1}\right] \\ &\quad-\sum_{h}\mathbb{1}_{h=b}d_{hjt}\left[\beta_h^1\log\ell_{hjt}-\beta_h^2\log\mu_{hjt-1}\right] \\ &\quad+\sum_{k,h,t}\mathbb{1}_{k=a}\mathbb{1}_{h=b}d_{kjt}d_{hjt}\left[\delta\log\frac{\tilde{w}_{kjt-1}}{\tilde{w}_{hjt-1}}+c_{kht}\right]+\eta_{abjt} \end{aligned} \tag{A.20}$$

where $\beta_k^1\equiv(\rho_k-1)$, $\beta_k^2\equiv\delta(\rho_k-1)$, and d_{kjt} is an indicator variable which equals 1 if firm j employs labor type k in periods t and $t-1$. This is similar to a "multivariate" regression where all the same regressors appear on the RHS of every equation. We now have $2K+1$ parameters to estimate, and thus need $2K+1$ instruments. Here we use lagged labor μ_{kjt-1}, lagged wages w_{kjt-1}, plus squares of both, giving us an overidentified system which we estimate using linear GMM (essentially 2SLS). Note that this approach allows for arbitrary cross-equation patterns of correlation between the error terms η_{abjt}.

Appendix B. Proofs of the main text results

B.1. Proof of Theorem 1. Fixed point representation of the existence of an equilibrium.

Recall that Assumptions 1, and 2, the optimal wage (eq 2.7) can be equivalently rewritten as

$$w_{kj} = \lambda_j F_k^j(\ell_{\cdot j}(w)) \frac{\mathcal{E}_{kj}(w)}{1+\mathcal{E}_{kj}(w)} \equiv B_{kj}(w), \quad \forall (k,j) \in \mathcal{K} \times \mathcal{J}. \tag{B.1}$$

Let $B(w) \equiv (B_{11}(.), ..., B_{KJ}(.))$. With this representation, showing the existence of an equilibrium matching is equivalent to show that the mapping $B(w)$ admits at least a fixed point, i.e. w^{eq}, such that $B(w^{eq}) = w^{eq}$ and then $s_{kj}(w^{eq}) = \frac{\partial G_{k\cdot}(v_{k\cdot})}{\partial v_{kj}}|_{v_{kj}=v_{kj}^{eq}}$ where $v_{kj}^{eq} \equiv \beta_{kj} \ln w_{kj}^{eq} + \ln u_{kj}$.

Let $\mathbb{T}_0 = \{w : 0 \leq w_{11} \leq \bar{\lambda}\bar{F}', ..., 0 \leq w_{KJ} \leq \bar{\lambda}\bar{F}'\}$, be a closed and bounded rectangular region in $\mathbb{R}^{KJ}$.

Step 0: Let $\underline{\xi}^t = (\underline{\xi}_1^t, ..., \underline{\xi}_{I+J}^t)$ and $\overline{\xi}^t = (\overline{\xi}_1^t, ..., \overline{\xi}_{I+J}^t)$ be vectors of arbitrarily small non-negative constants such that $\underline{\xi}_{kj}^t \leq w \leq \bar{\lambda}\bar{F}' - \overline{\xi}_{kj}^t$ for all $(k,j) \in \mathcal{K} \times \mathcal{J}$. $\underline{\xi}^t$ is chosen such that some of those components are strictly positive, which is ensured by the fact that under Assumptions 1, and 2, $\mathcal{C}^j \neq \{\emptyset\}$ for each $j \in J$. And define, $\mathbb{T}_\xi^t = \{w : \underline{\xi}_{11}^t \leq w_{11} \leq \bar{\lambda}\bar{F}' - \overline{\xi}_{11}^t, ..., \underline{\xi}_{KJ}^t \leq w_{KJ} \leq \bar{\lambda}\bar{F}' - \overline{\xi}_{KJ}^t\}$. Under Assumptions 1, and 2, also given that $B_{kj}(w)$ are continuous functions on a compact set $\mathbb{T}_\xi^t$ and $\lambda_j < \bar{\lambda}$, there exist vectors of non-negative constants (some strictly positive) $\underline{\eta}^t = (\underline{\eta}_{11}^t, ..., \underline{\eta}_{KJ}^t)$ and $\overline{\eta}^t = (\overline{\eta}_{11}^t, ..., \overline{\eta}_{KJ}^t)$ such that $\underline{\eta}_{kj}^t \leq B_{kj}(w) \leq \bar{\lambda}\bar{F}' - \overline{\eta}_{kj}^t$ for all $(k,j) \in \mathcal{K} \times \mathcal{J}$. More precisely, just take $\underline{\eta}_{kj}^t = \inf_{w \in \mathbb{T}_\xi^t} B_{kj}(w)$, and $\overline{\eta}_{kj}^t = \bar{\lambda}\bar{F}' - \sup_{w \in \mathbb{T}_\xi^t} B_{kj}(w)$, for all $(k,j) \in \mathcal{K} \times \mathcal{J}$.

Step 1: Define $\underline{\xi}_i^{t+1} = \min(\underline{\xi}_i^t, \underline{\eta}_i^t)$ for for $i = 11, ..., KJ$ and $\overline{\xi}_i^{t+1} = \min(\overline{\xi}_i^t, \overline{\eta}_i^t)$ for $i = 11, ..., KJ$.

Step 2: If $\underline{\xi}_i^{t+1} = \underline{\xi}_i^t$ and $\overline{\xi}_i^{t+1} = \overline{\xi}_i^t$ then stop the iteration and define $\underline{\epsilon}_i = \underline{\xi}_i^{t+1}$, $\bar{\epsilon}_i = \overline{\xi}_i^{t+1}$.

Step 3: If $\underline{\xi}_i^{t+1} \neq \underline{\xi}_i^t$ or $\overline{\xi}_i^{t+1} \neq \overline{\xi}_i^t$ then $t \leftarrow t+1$ and go back to step 0.

By construction $\underline{\xi}_i^t$ and $\overline{\xi}_i^t$ are decreasing positive sequences bounded from below by 0 then converge. So, when the iteration will stop in **Step 2**, let $\mathbb{T}_\epsilon = \{w : \underline{\epsilon}_{11} \leq w_{11} \leq \bar{\lambda}\bar{F}' - \bar{\epsilon}_{11}, ..., \underline{\epsilon}_{KJ} \leq w_{KJ} \leq \bar{\lambda}\bar{F}' - \bar{\epsilon}_{KJ}\}$ be a closed and bounded rectangular region in $\mathbb{R}^{KJ}$.

$B(w)$ is a continuously differentiable mapping such that $B(w)$: $\mathbb{T}_\epsilon \to \mathbb{T}_\epsilon$. Thus, the existence of a wage equilibrium w^{eq} exists by invoking the Brouwer fixed-point theorem. And then by construction we have the existence of (s^{eq}, w^{eq}).

B.2. Proof of Theorem 2. Let's define

$$\delta_{kj}(w) \equiv w_{kj} - \lambda_j F_k^j(\ell_{\cdot j}(w)) \frac{\mathcal{E}_{kj}(w)}{1+\mathcal{E}_{kj}(w)}, \quad \forall (k,j) \in \mathcal{K} \times \mathcal{J}. \tag{B.2}$$

$\delta(w) = (\delta_{11}(w), ..., \delta_{KJ}(w)) : \mathbb{T}_\epsilon \subseteq \mathbb{R}^{KJ} \longrightarrow \mathbb{R}^{KJ}$. Theorem 1 shows that an equilibrium is exist, showing the uniqueness is equivalent to show the global univalence of the mapping $\delta(w)$. Under Assumptions 1, and 2, $\delta(w)$ is continuously differentiable. Let $\mathbb{J}_\delta(w)$ be its Jacobian matrix, $\underset{(KJ \times KJ)}{\mathbb{J}_\delta(w)} = \begin{pmatrix} \frac{\partial \delta_{11}}{\partial w_{11}} & \cdots & \frac{\partial \delta_{11}}{\partial w_{KJ}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \delta_{KJ}}{\partial w_{11}} & \cdots & \frac{\partial \delta_{KJ}}{\partial w_{KJ}} \end{pmatrix}$.

According Gale and Nikaido (1965)'s result we know that $\delta(w)$ is globally univalent on $\mathbb{T}_\epsilon$ if $\mathbb{J}_\delta(w)$ is a P-matrix for all $w \in \mathbb{T}_\epsilon$. In the rest of the proof we will show that $\mathbb{J}_\delta(w)$ is indeed a P-matrix whenever Assumption 3 holds.

In the following we will make use extensive use of the following lemma:

Lemma 1. *Under Assumption 1, the following shape restrictions hold:*

$$\frac{\partial s_{kj}}{\partial w_{kl}} \begin{cases} \geq 0, & \text{if } l = j \\ \leq 0, & \text{if } l \in \mathcal{J}_0 \setminus \{j\} \end{cases}$$

Proof.

$$\begin{aligned} s_{kj} &= \mathbb{P}\big(v_{kj} + \epsilon_{ij} \geq v_{kj'} + \epsilon_{ij'} \text{ for all } j' \in \mathcal{J} \cup \{0\} \equiv \mathcal{J}_0\big) \\ &= \mathbb{P}\left(\underbrace{\epsilon_{i0} - \epsilon_{ij}}_{\varepsilon_{ij0}} \leq v_{kj} - v_{k0}, ..., \underbrace{\epsilon_{iJ} - \epsilon_{ij}}_{\varepsilon_{ijJ}} \leq v_{kj} - v_{kJ}\right) \\ &= F_{\varepsilon_{ij0},...,\varepsilon_{ijJ}}(v_{kj} - v_{k0}, ..., v_{kj} - v_{kJ}). \end{aligned}$$

Let $F^{(l)}_{X1,...,X_J}(x_1, ..., x_J) \equiv \frac{\partial}{\partial x_l} F_{X1,...,X_J}(x_1, ..., x_J)$. We have then:

$$\begin{aligned} \frac{\partial s_{kj}}{\partial v_{kl}} &= -F^{(l)}_{\varepsilon_{ij0},...,\varepsilon_{ijJ}}(v_{kj} - v_{k0}, ..., v_{kj} - v_{kJ}) \leq 0, \text{ for } l \neq j, \\ \frac{\partial s_{kj}}{\partial v_{kj}} &= \sum_{l \neq j} F^{(l)}_{\varepsilon_{ij0},...,\varepsilon_{ijJ}}(v_{kj} - v_{k0}, ..., v_{kj} - v_{kJ}) \geq 0, \end{aligned}$$

where both inequalities hold because $F_{\varepsilon_{ij0},...,\varepsilon_{ijJ}}(.)$ is a joint CDF.

□

Definition 2. *Let A be a real square matrix. (i) A is a P-matrix if every principal minor of A is positive, i.e. > 0. (ii) A is said to be a* **positive diagonally dominant** *matrix if there exists a strictly positive vector $d = (d_1, ..., d_n)$ where each $d_i > 0$ such that $d_i A_{ii} > \sum_{j \neq i} d_j |A_{ij}|$.*

According Proposition 1(ii) of Parthasarathy (1983, p.10) any real square matrix that is positive diagonally dominant is a P-matrix. Recall that under Assumption 2, $\mathcal{C}^j \neq \{\emptyset\}$, in fact in our modelling approach $\lambda_j F^j_k(\ell_{\cdot j}(w)) \frac{\mathcal{E}_{kj}(w)}{1+\mathcal{E}_{kj}(w)} = 0 \iff F^j_k(\ell_{\cdot j}(w)) = 0$ for all $w \in \mathbb{T}_\epsilon$, but according Assumption 2, for each $j \in \mathcal{J}$ there exists at least some k such that $F^j_k(\ell_{\cdot j}(w)) > 0$ then $\lambda_j F^j_k(\ell_{\cdot j}(w)) \frac{\mathcal{E}_{kj}(w)}{1+\mathcal{E}_{kj}(w)} > 0$. Under Assumptions 1, and 2, for all $k \in \mathcal{C}^j$ and $j \in \mathcal{J}$, we have

$$\frac{\partial \delta_{kj}}{\partial w_{ml}} = \begin{cases} 1 - \lambda_j \frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kj}} F^j_{kk}(\ell_{\cdot j}(w)) \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})} - \lambda_j F^j_k(\ell_{\cdot j}(w)) \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))^2} \frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial w_{kj}}, & \text{if } m = k, l = j \\ -\lambda_j \frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kl}} F^j_{kk}(\ell_{\cdot j}(w)) \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})} - \lambda_j F^j_k(\ell_{\cdot j}(w)) \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))^2} \frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial w_{kl}}, & \text{if } m = k, l \neq j \\ -\lambda_j \frac{\partial \ell_{mj}(w_{m\cdot})}{\partial w_{ml}} F^j_{km}(\ell_{\cdot j}(w)) \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})}, & \text{if } m \neq k. \end{cases}$$

for all $(m, l) \in \mathcal{K} \times \mathcal{J}$. Notice that for all $k \in \overline{\mathcal{C}^j} \equiv \mathcal{K} \setminus \mathcal{C}^j, j \in \mathcal{J}$, because $F^j_k(\ell_{\cdot j}(w)) = 0$ we have $\frac{\partial \delta_{kj}}{\partial w_{kj}} = 1$ and $\frac{\partial \delta_{kj}}{\partial w_{ml}} = 0$ for $m \neq k$ or $l \neq j$. For all $k \in \mathcal{C}^j$ denote $d_{kj} \equiv w_{kj}/\beta_{kj} > 0$ and for all $k \in \overline{\mathcal{C}^j}$ $d_{kj} = 1$ and this for all $j \in \mathcal{J}$. Let consider two cases:

Case 1: Assumption 3 holds: Under Assumption 3 we have the following sign restriction on $\frac{\partial \delta_{kj}}{\partial w_{ml}}$:

$$\frac{\partial \delta_{kj}}{\partial w_{ml}} = \begin{cases} 1 - \lambda_j \underbrace{\frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kj}}}_{\geq 0} \underbrace{F^j_{kk}(\ell_{\cdot j}(w))}_{\leq 0} \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})} - \lambda_j F^j_k(\ell_{\cdot j}(w)) \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))^2} \underbrace{\frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial w_{kj}}}_{\leq 0} > 0, & \text{if } m = k, l = j \\ -\lambda_j \underbrace{\frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kl}}}_{\leq 0} \underbrace{F^j_{kk}(\ell_{\cdot j}(w))}_{\leq 0} \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})} - \lambda_j F^j_k(\ell_{\cdot j}(w)) \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))^2} \underbrace{\frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial w_{kl}}}_{\geq 0} \leq 0, & \text{if } m = k, l \neq j, \\ -\lambda_j \frac{\partial \ell_{mj}(w_{m\cdot})}{\partial w_{ml}} \underbrace{F^j_{km}(\ell_{\cdot j}(w))}_{=0} \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})} = 0, & \text{if } m \neq k. \end{cases}$$

Therefore, for all $k \in \mathcal{C}^j$ and $j \in \mathcal{J}$, we can show that

$$\begin{aligned} &\frac{w_{kj}}{\beta_{kj}} \frac{\partial \delta_{kj}}{\partial w_{kj}} - \sum_{m \neq k \text{ or } l \neq j} \frac{w_{ml}}{\beta_{ml}} \left| \frac{\partial \delta_{kj}}{\partial w_{ml}} \right| = \\ &\quad \underbrace{\frac{w_{kj}}{\beta_{kj}}}_{>0} - \lambda_j \underbrace{\left[\frac{w_{kj}}{\beta_{kj}} \frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kj}} + \sum_{l \neq j} \frac{w_{kl}}{\beta_{kl}} \frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kl}} \right]}_{m_k \sum_{l \in \mathcal{J}} \frac{\partial s_{kj}(w_{k\cdot})}{\partial v_{kl}} = -m_k \frac{\partial s_{kj}(w_{k\cdot})}{\partial v_{k0}} \geq 0} \underbrace{F^j_{kk}(\ell_{\cdot j}(w))}_{\leq 0} \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})} \\ &\quad - \lambda_j \underbrace{F^j_k(\ell_{\cdot j}(w))}_{\geq 0} \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))^2} \underbrace{\left[\frac{w_{kj}}{\beta_{kj}} \frac{\partial \mathcal{E}_{kj}}{\partial w_{kj}} + \sum_{l \neq j} \frac{w_{kl}}{\beta_{kl}} \frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial w_{kl}} \right]}_{\sum_{l \in \mathcal{J}} \frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial v_{kl}} = -\frac{\partial \mathcal{E}_{kj}}{\partial v_{k0}} \leq 0} > 0. \end{aligned} \tag{B.3}$$

All the sign restrictions hold under Assumption 3 holds. Two main non-obvious points in the previous inequality are the following equalities $\sum_{l \in \mathcal{J}_0} \frac{\partial s_{kj}(w_{k\cdot})}{\partial v_{kl}} = 0$ and $\sum_{l \in \mathcal{J}_0} \frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial v_{kl}} = 0$. The trick behind these equalities is the fact that an increase of all mean gross utility $v_{k\cdot}$ does not affect the share s_{kj} as remarked by Berry (1994, page 267). The same argument applies also to the elasticity which justifies the second equality. Moreover for all $k \in \overline{\mathcal{C}^j}$, and $j \in \mathcal{J}$, $d_{kj} \frac{\partial \delta_{kj}}{\partial w_{kj}} - \sum_{m \neq k \text{ or } l \neq j} d_{ml} \left| \frac{\partial \delta_{kj}}{\partial w_{ml}} \right| > 0$ trivially holds. Therefore, $\mathbb{J}_\delta(w)$ is indeed a P-matrix for all $w \in \mathbb{T}_\epsilon$, and then $\delta(w)$ is globally univalent on $\mathbb{T}_\epsilon$, which complete the proof.

Case 2: Assumption 3 (i) holds: In such a context we can show that

$$\begin{aligned} &\frac{w_{kj}}{\beta_{kj}} \frac{\partial \delta_{kj}}{\partial w_{kj}} - \sum_{m \neq k \text{ or } l \neq j} \frac{w_{ml}}{\beta_{ml}} \left| \frac{\partial \delta_{kj}}{\partial w_{ml}} \right| = \\ &\quad \frac{w_{kj}}{\beta_{kj}} + \lambda_j \sum_{m \neq k} \underbrace{\left[-\frac{w_{mj}}{\beta_{mj}} \frac{\partial \ell_{mj}(w_{m\cdot})}{\partial w_{mj}} + \sum_{l \neq j} \frac{w_{ml}}{\beta_{ml}} \frac{\partial \ell_{mj}(w_{m\cdot})}{\partial w_{ml}} \right]}_{-\frac{\partial \ell_{mj}}{\partial v_{m0}} - 2 \frac{\partial \ell_{mj}}{\partial v_{mj}}} \left| F^j_{km}(\ell_{\cdot j}(w)) \right| \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})} \\ &\quad - \lambda_j \underbrace{\left[\frac{w_{kj}}{\beta_{kj}} \frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kj}} + \sum_{l \neq j} \frac{w_{kl}}{\beta_{kl}} \frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kl}} \right]}_{m_k \sum_{l \in \mathcal{J}} \frac{\partial s_{kj}(w_{k\cdot})}{\partial v_{kl}} = -m_k \frac{\partial s_{kj}(w_{k\cdot})}{\partial v_{k0}} \geq 0} \underbrace{F^j_{kk}(\ell_{\cdot j}(w))}_{\leq 0} \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})} \\ &\quad - \lambda_j \underbrace{F^j_k(\ell_{\cdot j}(w))}_{\geq 0} \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))^2} \underbrace{\left[\frac{w_{kj}}{\beta_{kj}} \frac{\partial \mathcal{E}_{kj}}{\partial w_{kj}} + \sum_{l \neq j} \frac{w_{kl}}{\beta_{kl}} \frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial w_{kl}} \right]}_{\sum_{l \in \mathcal{J}} \frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial v_{kl}} = -\frac{\partial \mathcal{E}_{kj}}{\partial v_{k0}} \leq 0}. \end{aligned}$$

Notice that the second term after the equality holds because, as discussed earlier, we have $\sum_{l\in\mathcal{J}} \frac{\partial s_{mj}(w_{m\cdot})}{\partial v_{ml}} = -\frac{\partial s_{mj}(w_{m\cdot})}{\partial v_{m0}}$. Therefore, we can write:

$$\frac{w_{kj}}{\beta_{kj}}\frac{\partial \delta_{kj}}{\partial w_{kj}} - \sum_{m\neq k \text{ or } l\neq j} \frac{w_{ml}}{\beta_{ml}}\left|\frac{\partial \delta_{kj}}{\partial w_{ml}}\right| = \frac{w_{kj}}{\beta_{kj}} + \lambda_j\Bigg\{ -\sum_{m\neq k}\left[\frac{\partial \ell_{mj}(w_{m\cdot})}{\partial v_{m0}} + 2\frac{\partial \ell_{mj}(w_{m\cdot})}{\partial v_{mj}}\right]\left|F^j_{km}(\ell_{\cdot j}(w))\right|$$
$$+\frac{\partial \ell_{kj}(w_{k\cdot})}{\partial v_{k0}}F^j_{kk}(\ell_{\cdot j}(w)) + F^j_k(\ell_{\cdot j}(w))\frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))\mathcal{E}_{kj}(w_{k\cdot})}\frac{\partial \mathcal{E}_{kj}}{\partial v_{k0}}\Bigg\} \times \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})}.$$

As can be seen, without additive separability in the production function the equilibrium can be unique if the RHS of the latter equality is positive. A sufficient condition for it is that

$$\Bigg\{ -\sum_{m\neq k}\left[\frac{\partial \ell_{mj}(w_{m\cdot})}{\partial v_{m0}} + 2\frac{\partial \ell_{mj}(w_{m\cdot})}{\partial v_{mj}}\right]\left|F^j_{km}(\ell_{\cdot j}(w))\right| + \frac{\partial \ell_{kj}(w_{k\cdot})}{\partial v_{k0}}F^j_{kk}(\ell_{\cdot j}(w)) + \\ + F^j_k(\ell_{\cdot j}(w))\frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))\mathcal{E}_{kj}(w_{k\cdot})}\frac{\partial \mathcal{E}_{kj}}{\partial v_{k0}}\Bigg\} \geq 0 \tag{B.4}$$

for all $w \in \mathbb{T}_\epsilon$.

B.3. **Proof of Proposition 1.**

Lemma 2. *Under Assumptions 1, 2, and 3, $\delta(w)$ is generalized nonlinear diagonally dominant on $\mathbb{T}_\epsilon$.*

Proof. All partial derivative of $\delta(w)$ exists and are continuous. Let's $\mathbb{J}_\delta(w) \equiv \delta(w)'$ be its Jacobian matrix which is continuous on $\mathbb{T}_\epsilon$. $\delta(w)$ is Frèchet-differentiable on $\mathbb{T}_\epsilon$ then it is Gâteaux-differentiable on $\mathbb{T}_\epsilon$ which is a convex compact subset of $\mathbb{R}_{KJ}$. In the case 1 of the Proof of Theorem 2, we show that $\mathbb{J}_\delta(w)$ is *a generalized diagonally dominant matrix* in the language of Gan et al. (2006) and this for all $w \in \mathbb{T}_\epsilon$. The proof is complete once we invoke Theorem 8 of Gan et al. (2006).

□

Lemma 3. *Under Assumptions 1, 2, and 3,*

For any $w \in \mathbb{T}_\epsilon$, and $kj = 1, ..., KJ$ the following equation in x_{kj}:
$\psi(x_{kj}, w_{-kj}) \equiv \delta_{kj}(w_{11}, ..., w_{1J}, ..., w_{k,j-1}, x_{kj}, w_{k,j+1}, ..., w_{KJ}) = 0$ as a unique solution x^*_{kj}.

Proof. In the case 1 of the Proof of Theorem 2, we show that $\frac{\partial \psi(x_{kj}, w_{-kj})}{\partial x_{ij}} \geq 1 > 0$, then $\psi(x_{kj}, w_{-kj})$ is strictly increasing in x_{kj} for any $w_{-kj} \in \mathbb{T}_\epsilon$. In addition, as can be seen in the proof of Theorem 1, $\psi(\underline{\epsilon}_{kj}, w_{-kj}) \leq 0 \leq \psi(\bar{\lambda}\bar{F}' - \bar{\epsilon}_{kj}, w_{-kj})$ for for any $w_{-kj} \in \mathbb{T}_\epsilon$. This completes the proof. □

Under Assumptions 1, 2, and 3 Lemmata 2, and 3 hold, then we could invoke Theorem 18 of Frommer (1991). Remark that both underrelaxed Gauss-siedel and Jacobi iteration are special cases of the asynchronous iterative methods discussed in Frommer (1991) Theorem 18. This complete the Proof of Proposition 1.

B.4. **Proof of Proposition 2.** Under Assumptions 1, 2, and 3, we proved that we have an unique equilibrium w^{eq} such that $w^{eq} = B(w^{eq})$. For sake of simplicity let us ignore the upper-script eq in the rest of the proof. By the Implicit Function Theorem we have:

$$\frac{dw}{dw_{k0}} = \mathbb{J}_\delta^{-1}(w)\frac{\partial B(w)}{\partial w_{k0}},$$
$$\frac{dw}{d\gamma_{kl}} = \mathbb{J}_\delta^{-1}(w)\frac{\partial B(w)}{\partial \gamma_{kl}},$$
$$\frac{dw}{d\theta_l} = \mathbb{J}_\delta^{-1}(w)\frac{\partial B(w)}{\partial \theta_l}.$$

Under Assumption 3, $\mathbb{J}_\delta(w)$ is a block diagonal matrix, more precisely it can be written $\underset{(KJ\times KJ)}{\mathbb{J}_\delta(w)} = \begin{pmatrix} \mathbb{J}_{\delta,1\cdot}(w) & 0 & \cdots & 0 \\ 0 & \mathbb{J}_{\delta,2\cdot}(w) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbb{J}_{\delta,K\cdot}(w) \end{pmatrix}$, where $\underset{(J\times J)}{\mathbb{J}_{\delta,k\cdot}(w)} = \begin{pmatrix} \frac{\partial \delta_{k1}}{\partial w_{k1}} & \cdots & \frac{\partial \delta_{k1}}{\partial w_{kJ}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \delta_{kJ}}{\partial w_{k1}} & \cdots & \frac{\partial \delta_{kJ}}{\partial w_{kJ}} \end{pmatrix}$. The case 1 of the Proof of Theorem 2, shows that each $\mathbb{J}_{\delta,k\cdot}(w)$ for $k \in \mathcal{K}$ is positive diagonally dominant, therefore its inverse exists and then we have, $\underset{(KJ\times KJ)}{\mathbb{J}_\delta^{-1}(w)} = \begin{pmatrix} \mathbb{J}_{\delta,1\cdot}^{-1}(w) & 0 & \cdots & 0 \\ 0 & \mathbb{J}_{\delta,2\cdot}^{-1}(w) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbb{J}_{\delta,K\cdot}^{-1}(w) \end{pmatrix}$. We then have $\frac{dw_{m\cdot}}{dw_{k0}} = \mathbb{J}_{\delta,m\cdot}^{-1}(w)\frac{\partial B_{m\cdot}(w)}{\partial w_{k0}}$ where $w_{m\cdot} = \begin{pmatrix} w_{m1} \\ \vdots \\ w_{mJ} \end{pmatrix}$, and $B_{m\cdot}(w) = \begin{pmatrix} B_{m1}(w) \\ \vdots \\ B_{mJ}(w) \end{pmatrix}$. Our derived bounds come from the linear algebra results on M-matrices and inverse M-matrices, i.e. Carlson and Markham (1979); Fiedler and Pták (1962). In fact, case 1 of the Proof of Theorem 2, shows that any $\mathbb{J}_{\delta,k\cdot}(w)$ for $k \in \mathcal{K}$ is positive diagonally dominant and have non-positive off diagonal elements. Then, $\mathbb{J}_{\delta,k\cdot}(w)$, and $\mathbb{J}_\delta(w)$ are M Matrices. Our proofs widely use the result (4.2) of Fiedler and Pták (1962), which states that if A and B are two M matrices such that $A \leqq B$, then $A^{-1} \geqq B^{-1} \geqq 0$. Let's denote by DA the diagonal matrix formed by the diagonal elements of the matrix A. Under Assumption 3, we have $\mathbb{J}_{\delta,k\cdot}(w) \leq D\mathbb{J}_{\delta,k\cdot}(w) \Rightarrow \mathbb{J}_{\delta,k\cdot}^{-1}(w) \geq [D\mathbb{J}_{\delta,k\cdot}(w)]^{-1} \Rightarrow \mathbb{J}_{\delta,k\cdot}^{-1}(w)\frac{\partial B_{k\cdot}(w)}{\partial w_{k0}} \geq [D\mathbb{J}_{\delta,k\cdot}(w)]^{-1}\frac{\partial B_{k\cdot}(w)}{\partial w_{k0}}$ where the last inequality holds since $\frac{\partial B_{kj}(w)}{\partial w_{k0}} \geq 0$ under Assumption 3.

It follows from the latter inequality that:

$$\frac{\partial w_{kj}}{\partial w_{k0}} \geq \frac{w_{kj}}{w_{k0}}\frac{\psi_{k,j0}}{1-\psi_{k,jj}} \geq 0$$

where $\psi_{k,jl} = \left(\frac{w_{kl}}{\ell_{kj}}\frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kl}}\left(\frac{F^j_{kk}}{F^j_k}\ell_{kj}\right) + \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))}\frac{w_{kl}}{\mathcal{E}_{kj}(w_{k\cdot})}\frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial w_{kl}}\right)$. This latter inequality becomes evident as soon as you remark that: $\frac{\partial \delta_{kj}}{\partial w_{kl}}\begin{cases} -\left(\frac{w_{kj}}{w_{kl}}\right)\psi_{k,jl} \text{ if } j \neq l \\ 1-\psi_{k,jl} \text{ if } j = l \end{cases}$. This proves the first set of bounds.

Second, for $a_{ll} > 0$ and $a_{jl} \leq 0$ when $j \neq l$ it can be shown that

$$H^{-1}(a_{\cdot\cdot}) \equiv \begin{pmatrix} a_{11} & 0 & \cdots & 0 & a_{1l} & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & 0 & a_{ll} & 0 & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & a_{l+1,l+1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & a_{J,J} \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 1/a_{11} & 0 & \cdots & 0 & -a_{1l}/a_{11}a_{ll} & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & 0 & 1/a_{ll} & 0 & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & 1/a_{l+1,l+1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 1/a_{J,J} \end{pmatrix},$$

$\frac{\partial B_{k\cdot}(w)}{\partial \theta_l} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ B_{kl}(w)/\theta_l \\ 0 \\ \vdots \\ 0 \end{pmatrix} \geq 0$. For $a_{jl} \equiv \frac{\partial \delta_{kj}}{\partial w_{kl}}$ we have $\mathbb{J}_{\delta,k\cdot}(w) \leq H\left(\frac{\partial \delta_{k\cdot}}{\partial w_{k\cdot}}\right) \Rightarrow \mathbb{J}^{-1}_{\delta,k\cdot}(w) \geq \left[H\left(\frac{\partial \delta_{k\cdot}}{\partial w_{k\cdot}}\right)\right]^{-1} \Rightarrow$ $\mathbb{J}^{-1}_{\delta,k\cdot}(w)\frac{\partial B_{k\cdot}(w)}{\partial \theta_l} \geq \left[H\left(\frac{\partial \delta_{k\cdot}}{\partial w_{k\cdot}}\right)\right]^{-1}\frac{\partial B_{k\cdot}(w)}{\partial \theta_l}$. The latter inequality implies that fo $j \leq l$ we have:

$$\frac{\partial w_{kj}}{\partial \theta_l} \begin{cases} \geq -\frac{\frac{\partial \delta_{kj}}{\partial w_{kl}}}{\frac{\partial \delta_{kj}}{\partial w_{kj}}\frac{\partial \delta_{kl}}{\partial w_{kl}}}\frac{B_{kl}(w)}{\theta_l} = \frac{w_{kj}\psi_{k,jl}}{\theta_l(1-\psi_{k,jj})(1-\psi_{k,ll})} \geq 0 \text{ if } j < l \\ \geq \frac{1}{\frac{\partial \delta_{kl}}{\partial w_{kl}}}\frac{B_{kl}(w)}{\theta_l} = \frac{w_{kl}}{\theta_l(1-\psi_{k,ll})} > 0, \text{ if } j = l. \text{ otherwise.} \end{cases} \tag{B.5}$$

For $j < l$, we can follow the same process by considering H as a lower triangular matrix. The exact same proof holds for $\frac{\partial w_{kj}}{\partial \theta_l}$. This completes the proof.

Special case: Duopsony. In this special case, we could have a passthrough formula that will hold at equality. This will allow us to have an intuition of the shock transmission from a firm j to a firm l. Recall that $\frac{dw_{m\cdot}}{dw_{k0}} = \mathbb{J}^{-1}_{\delta,m\cdot}(w)\frac{\partial B_{m\cdot}(w)}{\partial w_{k0}}$, and $\frac{\partial \delta_{kj}}{\partial w_{kl}} = -\left(\frac{w_{kj}}{w_{kl}}\right)\psi_{k,jl}$ for $l \neq j$.

Now, consider that $\mathcal{J} = \{j, l\}$. In this special case the inverse of the Jacobian matrix is given by: $(\mathbb{J}_{\delta,k\cdot}(w))^{-1} = \begin{pmatrix} \frac{\partial \delta_{kj}}{\partial w_{kj}} & \frac{\partial \delta_{kj}}{\partial w_{kl}} \\ \frac{\partial \delta_{kl}}{\partial w_{kj}} & \frac{\partial \delta_{kl}}{\partial w_{kl}} \end{pmatrix}^{-1} = \frac{1}{(1-\psi_{k,jj})(1-\psi_{k,ll})-\psi_{k,jl}\psi_{k,lj}} \begin{pmatrix} (1-\psi_{k,ll}) & \left(\frac{w_{kj}}{w_{kl}}\right)\psi_{k,jl} \\ \left(\frac{w_{kl}}{w_{kj}}\right)\psi_{k,lj} & (1-\psi_{k,jj}) \end{pmatrix}$. Then we can easily derive the following:

$$\frac{w_{k0}}{w_{kj}}\frac{\partial w_{kj}}{\partial w_{k0}} = \frac{(1-\psi_{k,ll})\psi_{k,j0} + \psi_{k,jl}\psi_{k,l0}}{(1-\psi_{k,jj})(1-\psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \geq 0 \tag{B.6}$$

$$\frac{u_{kl}}{w_{kj}}\frac{\partial w_{kj}}{\partial u_{kl}} = \frac{(1-\psi_{k,ll})\phi_{k,jl} + \psi_{k,jl}\phi_{k,ll}}{(1-\psi_{k,jj})(1-\psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \gtreqless 0 \tag{B.7}$$

$$\frac{u_{kl}}{w_{kl}}\frac{\partial w_{kl}}{\partial u_{kl}} = \frac{(1-\psi_{k,jj})\phi_{k,ll} + \psi_{k,lj}\phi_{k,jl}}{(1-\psi_{k,jj})(1-\psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \gtreqless 0 \tag{B.8}$$

$$\frac{\theta_l}{w_{kj}}\frac{\partial w_{kj}}{\partial \theta_l} = \frac{\psi_{k,jl}}{(1-\psi_{k,jj})(1-\psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \geq 0 \tag{B.9}$$

$$\frac{\theta_l}{w_{kj}}\frac{\partial w_{kj}}{\partial \theta_l} = \frac{(1-\psi_{k,jj})}{(1-\psi_{k,jj})(1-\psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \geq 0 \tag{B.10}$$

where the signs restrictions hold, because $\psi_{k,jl}, \phi_{k,jl} \geq 0$ for $l \neq j$, and $\psi_{k,ll}, \phi_{k,ll} \leq 0$ with $\phi_{k,jl} = \left(\frac{u_{kl}}{\ell_{kj}}\frac{\partial \ell_{kj}(w_{k\cdot})}{\partial u_{kl}}\left(\frac{F^j_{kk}}{F^j_k}\ell_{kj}\right) + \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))}\frac{u_{kl}}{\mathcal{E}_{kj}(w_{k\cdot})}\frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial u_{kl}}\right)$

Appendix C. Data and Sample Description

Our data consists of several administrative registers provided by Statistics Denmark for the years 2001-2019. These include annual cross-section data from the Danish register-based, matched employer-employee dataset IDA (Integrated Database for Labor Market Research) and other annual datasets, divided into IDAN, IDAS, and IDAP. The datasets are linked by individual identifiers for persons, firms, and establishments. Table C.1 lists the relevant datasets and details.

Table C.1. Data Description (Datasets and Variables).

Category	**Register**	**Variables**
workers	IDAN (jobs yearly panel)	firm and establishment indicator, establishment location, yearly earnings, hours worked, share of the year worked, type of job (primary, secondary), type of job (part-time/full-time), type of job (occupation, DISCO code)
not employed	BEF (population register) IDAN	We classify as not employed all individuals in the relevant age groups who are not recorded in IDAN.
unemployed	IND (income dataset, individual yearly panel), IDAP (worker dataset, individual yearly panel)	unemployment benefits, duration of unemployment
firms and establishments	FIRM, IDAS (workplace panel)	firm revenue, sector of industry (5-digit industry classification based on NACE rev. 2), establishment location (municipality)
k-groups	UDDA (education panel), BEF (individual yearly panel)	age, highest acquired education, sex
commuting zones	Eckert et al. (2022) (available on Fabian Eckert website)	commuting zone (link to municipality)

We restrict the dataset to individuals between 26 and 60 years of age who work full-time as employees in the private sector whose job is linked to a physical establishment. We drop individuals employed in the financial sector; firms in the financial sector are not required to report revenue data and very few do. Details on data and sample selection are in table C.2. In total, our dataset consists of $12,742,746$ individual-year combinations. Our sample construction selects the data in a few important ways: The full population of salaried jobs in Denmark in 2001-2019 is 49.3 percent female. This goes down to 35.8 percent when we drop the public sector and further to 31.8 percent when we exclude the financial sector and non-full-time jobs. Workers in the private-sector with full-time jobs are on average one year older than the full worker population, and have average yearly earnings of $71,491$ USD, higher than the full-worker-population average of $42,867$ USD.

Table C.2. Worker Sample Selection.

	step	observations	share in public sector	share in financial sector	share full-time	share female	avg. age	avg. yearly earnings (2022 USD)
1	All salaried jobs in Denmark between 2001 and 2019	76,869,608						
2	Keep jobs held by workers in relevant k-groups	50,263,511	0.229	0.024	0.437	0.493	42.454	42,867
3	Keep jobs with market information (primary jobs)	32,486,151	0.355	0.037	0.648	0.487	42.964	56,389
4	Drop workers in small commuting zones	32,106,644	0.354	0.037	0.768	0.487	42.943	56,474
5	Drop jobs with no earnings or hours	32,094,227	0.354	0.037	0.648	0.487	42.944	56,493
6	Drop public sector jobs	20,719,775		0.057	0.660	0.358	42.482	59,641
7	Drop financial sector jobs	19,538,794			0.653	0.349	42.425	58,296
8	Keep full-time, highest-paying jobs	**12,742,746**				0.318	43.518	71,491
10	Only period 2004-2016	8,614,260						

Find a detailed description of the selection steps below:

(1) This step excludes self employed and employers, as well as their spouses if their main source of income is from assisting the spouse's enterprise; it includes all other types of jobs.
(2) This step drops workers not appearing in the population registers, younger and older workers, as well as workers with no education status recorded (this applies mostly to immigrant workers). Therefore, this step excludes jobs held by workers not resident in Denmark.
(3) This step drops jobs without real establishment code, i.e., all non-primary jobs and primary jobs with missing or fictitious establishment code. Primary jobs are the most important connection to the labor market (longest employment period and largest ATP payments). Workers with fictitious workplaces (establishment nr. = 0) are those who cannot be linked to any of the employer's registered workplaces, either because they work from home or in various workplaces (such as cleaners, home nurses). Workers with no workplace (establishment nr. = .) are those with multiple workplaces for which one unique workplace cannot be identified. In 2,491,168 instances, where the establishment information is missing only in one year during a continuous employment spell at the same firm, we impute it.
(4) Drop jobs in establishments in the islands of Christiansœ, Bornholm, Samsœ, and Æro.
(5) Drop jobs with no information on earnings or hours
(6) Drop if the sector of industry of the employer is one of the following nacee-2 codes {O,P,Q,T,U,X}.
(7) Drop if the sector of industry of the employer is nacee-2 code K (this sector has an extreme underreporting of revenue data).
(8) We define full-time jobs as jobs with weekly schedule of 30 hours or more.

We denote establishments with the subscript j, time (years) with the subscript t, and worker type (or k-groups) with the subscript k. Worker types are divided by sex (male or female) age group (26-35, 36-50, 51-60) and education level (completed or not tertiary education). We collapse the individual-level dataset at the (k, j, t) level leading to $4,487,628$ observations. We restrict the estimation dataset to only establishments that have no missing values for any of the key variables. Table C.3 details the sample selection process.

The key variables we use in the estimation are:

- w_{kjt}: mean earnings by k-group, establishment, year
- w_{k0t}: mean non-employment income by k-group, year

Table C.3. Establishment Sample Selection and Construction of the Estimation Dataset.

	step	total observations	unique establishments
1	collapse at the kgroup-establishment-year (k, j, t) level	4,487,628	259,195
2	merge revenue data (firm, year)	-	-
3	add share of non-employed/unemployed and average unemployment income	-	-
4	drop observations with wage bill to revenue ratio above 80% (drops all observations with missing revenue)	4,054,235	238,299
	keep observations for firms that appear at least once in the estimation dataset	**3,069,490**	**63,525**
5	create estimation variables	-	-
6	keep observations in 2004-2017 to accommodate for long run lags $(x_{jkt+2} - x_{jkt-3})$ and data break	2,268,523	-
7	drop firms/k-groups with not enough longevity to allow for computing short-run lags $(x_{jkt} - x_{jkt-1})$	2,318,335	-
8	drop firms/k-groups with not enough longevity to allow for computing long-run lags $(x_{jkt+2} - x_{jkt-3})$	1,914,366	-
9	drop firms employing only one k-group (necessary for the second instrument)	**1,101,541**	**63,525**

Start with panel of selected workers in years 2001-2019. Variables: full-time-equivalent, earnings, k-group (sex, age, education), local market (commuting zone, industry), firm, establishment, year (12,742,746 individuals).

- s_{kjt} and $s_{kj|gt}$: employment shares, by k-group, establishment, year, overall and by market g (inside shares)
- $s_{\sim kj|gt}$: sum of the inside shares for all other labor types employed by establishment j, by k-group, year, market
- R_{jt}: establishment-level revenue by year, obtained allocating firm revenue across establishments in proportion to their wage bills

Appendix D. Appendix Figures and Tables

Table D.1. Establishment characteristics, by commuting zone (full sample)

	n. unique estab.	n. estab. per firm		n. of workers per estab.		n. of k-groups per estab.		estab. revenue (1,000 UDS)		average wage (USD)	
commuting zone		mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.
1. North and East Zealand (Copenhagen)	92,763	1.225	3.535	8.454	40.626	2.685	2.356	6,171	61,366	65,148	36,819
2. West and South Zealand (Slagelse)	10,714	1.229	3.378	5.816	33.274	2.348	1.897	3,941	55,333	55,326	15,962
3. West and South Zealand (Køge)	11,953	1.205	3.255	5.715	19.334	2.383	1.929	3,533	22,430	55,888	17,122
4. West and South Zealand (Nykøbing Falster)	4,432	1.249	3.408	5.294	14.390	2.316	1.794	2,729	10,837	51,730	13,878
7. Fyn (Odense)	18,870	1.251	3.679	7.285	24.103	2.686	2.251	4,829	29,984	56,571	26,792
8. Fyn (Svendborg)	2,927	1.183	2.346	4.953	9.919	2.400	1.917	2,820	8,953	54,654	17,221
9. South Jutland (Sønderborg)	5,721	1.224	2.299	8.191	48.528	2.613	2.162	5,614	31,400	54,921	16,691
10. South Jutland (Ribe)	2,041	1.137	1.874	5.554	17.850	2.298	1.879	4,179	22,629	52,261	13,967
11. South Jutland (Kolding)	9,586	1.285	4.333	7.323	19.109	2.727	2.280	4,924	17,372	56,779	17,734
12. Mid-South Jutland (Vejle)	14,569	1.223	3.707	7.820	45.272	2.680	2.258	6,017	58,046	57,835	21,745
13. South-West Jutland (Esbjerg)	10,559	1.218	3.293	6.981	22.509	2.590	2.167	5,419	58,484	55,862	16,837
14. West Jutland (Herning)	9,536	1.233	3.943	7.040	22.462	2.605	2.156	4,583	22,913	55,664	15,332
15. North-West Jutland (Thisted)	2,135	1.172	2.080	6.329	21.196	2.416	1.975	4,009	15,606	54,166	13,972
16. East Jutland (Aarhus)	31,828	1.232	3.362	7.399	24.617	2.678	2.271	5,160	53,258	59,101	22,934
17. Mid-North Jutland (Viborg)	7,988	1.169	2.632	6.901	47.707	2.493	2.077	4,071	26,117	54,906	15,958
19. North Jutland (Aalborg)	23,573	1.232	3.772	6.523	21.000	2.520	2.115	4,499	49,905	55,542	18,252
All of Denmark	259,195	1.227	3.494	7.414	33.071	2.611	2.223	5,198	49,994	59,311	27,048

Source: Administrative registers, Statistics Denmark. Full population of private sector establishments in Denmark (step 1 in table C.3). Commuting zones computed for 2005 by Eckert et al. (2022), largest city in parentheses. We drop six small islands and we merge Aalborg and Frederikshavn. Revenue and average wage at the firm in 2022 USD.

Table D.2. Establishment characteristics, by commuting zone (estimation sample, all years)

	n. unique estab.	n. estab. per firm		n. of workers per estab.		n. of k-groups per estab.		estab. revenue (1,000 UDS)		average wage (USD)	
commuting zone		mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.
1. North and East Zealand (Copenhagen)	20,358	1.204	3.052	13.603	54.919	3.672	2.581	11,148	71,722	68,275	24,494
2. West and South Zealand (Slagelse)	2,586	1.257	4.753	9.008	46.099	3.106	2.087	6,915	77,428	57,408	13,925
3. West and South Zealand (Køge)	2,827	1.205	2.702	9.000	26.336	3.212	2.113	6,138	31,361	58,367	15,116
4. West and South Zealand (Nykøbing Falster)	1,099	1.272	3.142	7.981	18.888	3.028	1.954	4,528	14,376	53,726	12,963
7. Fyn (Odense)	4,904	1.220	2.536	11.125	31.530	3.575	2.438	7,969	39,128	58,870	16,999
8. Fyn (Svendborg)	751	1.146	1.402	7.356	12.324	3.189	2.086	4,691	11,938	56,857	14,928
9. South Jutland (Sønderborg)	1,554	1.238	3.337	12.882	65.503	3.433	2.352	9,352	41,824	57,073	14,425
10. South Jutland (Ribe)	512	1.139	1.351	9.010	24.495	3.129	2.118	7,210	31,337	54,684	12,545
11. South Jutland (Kolding)	2,636	1.263	2.919	11.245	24.572	3.613	2.485	8,000	22,656	59,639	16,070
12. Mid-South Jutland (Vejle)	3,934	1.209	2.927	12.184	60.546	3.587	2.456	10,022	78,264	60,382	18,023
13. South-West Jutland (Esbjerg)	2,915	1.207	2.043	10.648	28.233	3.445	2.362	8,952	79,013	58,512	15,138
14. West Jutland (Herning)	2,672	1.199	3.521	10.817	29.165	3.464	2.344	7,453	30,365	57,933	13,439
15. North-West Jutland (Thisted)	585	1.205	3.829	9.958	28.068	3.217	2.174	6,650	20,664	56,651	12,735
16. East Jutland (Aarhus)	8,203	1.248	3.359	11.303	31.179	3.588	2.456	8,625	72,640	61,478	16,908
17. Mid-North Jutland (Viborg)	2,092	1.191	4.713	10.737	65.201	3.349	2.254	6,614	28,985	57,435	15,628
19. North Jutland (Aalborg)	5,897	1.236	3.810	10.202	27.552	3.412	2.330	7,560	67,593	57,936	16,245
All of Denmark	63,525	1.219	3.240	11.591	44.041	3.515	2.427	8,909	62,962	61,787	19,573

Source: Administrative registers, Statistics Denmark. Restricted sample of establishments with no missing values for the key estimation variables (step 5 in table C.3). Commuting zones computed for 2005 by Eckert et al. (2022), largest city in parentheses. We drop six small islands and we merge Aalborg and Frederikshavn. Revenue and average wage at the firm in 2022 USD.

Table D.3. Establishment characteristics, by industry (full sample)

	n. unique estab.	n. estab. per firm		n. of workers per estab.		n. of k-groups per estab.		estab. revenue (1,000 UDS)		average wage (USD)	
commuting zone		mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.
A. Agriculture, forestry, and fishery	13,486	1.042	0.711	2.302	4.045	1.643	1.246	1,720	2,909	48,810	13,767
B. Mining and quarrying	425	1.690	3.088	13.872	62.899	2.767	2.500	35,212	298,783	72,555	101,517
C. Manufacturing	20,937	1.171	1.237	18.924	73.662	3.872	2.978	12,355	73,817	60,794	18,468
D. Electricity, gas, steam etc.	925	1.267	2.041	15.340	46.974	3.372	2.926	34,650	321,543	73,488	30,898
E. Water supply, sewerage etc.	1,957	2.129	3.316	10.479	21.034	3.112	2.306	4,353	14,119	59,114	13,886
F. Construction	31,967	1.050	0.738	5.145	14.408	2.298	1.696	2,649	12,075	57,610	14,378
G. Wholesale and retail trade	69,193	1.383	5.722	5.514	15.559	2.518	1.992	6,679	36,576	56,683	21,619
H. Transportation	15,570	1.274	5.125	11.277	50.020	2.794	2.331	7,666	114,439	57,890	25,777
I. Accommodation and food services	15,780	1.239	3.003	3.370	9.242	2.038	1.638	1,488	4,217	48,049	13,443
J. Information and communication	15,495	1.182	3.108	10.968	49.839	2.912	2.604	5,163	29,492	76,131	40,250
L. Real estate	13,050	1.344	2.311	3.541	8.919	2.080	1.728	1,139	4,435	59,727	25,909
M. Knowledge-based services	27,463	1.136	1.231	7.589	30.830	2.753	2.433	2,798	18,008	72,659	47,190
N. Travel agent, cleaning etc.	13,831	1.290	2.322	6.724	19.534	2.668	2.325	3,153	12,084	59,338	36,342
R. Arts, entertainment, recreation	5,804	1.395	2.842	5.765	14.060	2.799	2.420	1,048	22,416	54,942	19,372
S. Other services	13,312	1.126	1.471	4.523	13.985	2.222	1.972	419	2,547	55,563	16,467
All industries	259,195	1.227	3.494	7.414	33.071	2.611	2.223	5,198	49,994	59,311	27,048

Source: Administrative registers, Statistics Denmark. Full population of private sector establishments in Denmark (step 1 in table C.3). 5-digit industry classification based on NACE rev. 2. We exclude the public sector, including the health and education sectors. Revenue and average wage at the firm in 2022 USD.

Table D.4. Establishment characteristics, by industry (estimation sample, all years)

	n. unique estab.	n. estab. per firm		n. of workers per estab.		n. of k-groups per estab.		estab. revenue (1,000 UDS)		average wage (USD)	
commuting zone		mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.
A. Agriculture, forestry, and fishery	2,238	1.046	0.581	3.782	5.724	2.372	1.583	2,840	4,262	50,896	11,758
B. Mining and quarrying	134	1.625	2.554	17.891	68.175	3.553	2.662	48,419	357,164	70,750	31,235
C. Manufacturing	8,850	1.171	1.232	24.457	84.925	4.632	2.998	16,004	84,926	61,765	14,128
D. Electricity, gas, steam etc.	306	1.178	1.061	20.179	57.205	4.095	3.001	64,549	456,296	74,386	33,991
E. Water supply, sewerage etc.	438	1.564	1.940	12.983	25.803	3.719	2.517	7,940	19,440	60,679	12,300
F. Construction	8,741	1.059	0.753	7.549	17.836	3.005	1.814	3,949	15,334	59,681	12,588
G. Wholesale and retail trade	21,282	1.356	4.931	7.916	19.278	3.254	2.156	9,794	45,456	59,433	19,340
H. Transportation	4,307	1.322	5.440	16.608	61.499	3.644	2.484	11,221	105,985	59,758	18,708
I. Accommodation and food services	2,018	1.210	1.995	5.749	14.367	3.018	2.008	2,709	6,364	51,705	12,648
J. Information and communication	3,498	1.212	2.826	18.323	63.986	4.175	2.851	8,926	38,975	76,905	24,703
L. Real estate	1,613	1.174	1.121	5.155	12.528	2.922	2.013	2,714	7,538	68,106	28,961
M. Knowledge-based services	6,086	1.160	1.111	12.136	40.652	3.912	2.622	4,899	24,755	72,717	24,334
N. Travel agent, cleaning etc.	2,704	1.182	1.136	7.918	23.035	3.305	2.284	5,492	17,011	62,117	20,404
R. Arts, entertainment, recreation	386	1.085	0.687	9.414	20.773	3.896	2.719	8,397	70,779	60,055	17,245
S. Other services	924	1.144	1.297	7.322	16.345	2.951	2.251	2,260	5,253	57,559	16,769
All industries	63,525	1.219	3.240	11.591	44.041	3.515	2.427	8,909	62,962	61,787	19,573

Source: Administrative registers, Statistics Denmark. Restricted sample of establishments with no missing values for the key estimation variables (step 5 in table C.3). 5-digit industry classification based on NACE rev. 2. We exclude the public sector, including the health and education sectors. Revenue and average wage at the firm in 2022 USD.

TABLE D.5. Labor Supply Parameter Estimates Across k-groups

		IV			OLS		
		β_k	σ_{kg}		β_k	σ_{kg}	
	k-group (k)		CZ 1 (CPH)	Avg. across CZ		CZ 1 (CPH)	Avg. across CZ
1	Female, 26-35, no college	1.701	3.966	3.228	-0.002	5.548	4.342
		[1.386; 2.0786]	[3.014; 4.445]		[-0.020; 0.013]	[4.359; 5.715]	
2	Female, 26-35, college	1.922	5.698	2.803	-0.099	6.352	3.405
		[1.315; 2.5072]	[3.168; 7.324]		[-0.124; -0.072]	[4.627; 6.382]	
3	Male, 26-35, no college	1.392	5.654	3.800	0.321	6.560	4.240
		[1.377; 1.5974]	[4.043; 5.863]		[0.308; 0.325]	[5.179; 6.146]	
4	Male, 26-35, college	2.225	3.926	3.923	0.323	5.057	3.423
		[1.823; 2.6060]	[2.758; 4.612]		[0.305; 0.341]	[3.857; 5.056]	
5	Female, 36-50, no college	1.078	6.169	3.913	0.226	6.347	3.991
		[0.997; 1.3178]	[4.385; 6.516]		[0.216; 0.229]	[4.769; 5.978]	
6	Female, 36-50, college	1.540	4.463	3.776	0.000	4.813	3.657
		[1.234; 2.1316]	[2.946; 5.052]		[0.049 ; 0.079]	[3.565; 4.559]	
7	Male, 36-50, no college	0.874	6.545	3.930	0.272	6.351	4.241
		[0.917; 1.0248]	[4.586; 6.043]		[0.263; 0.274]	[4.680; 5.461]	
8	Male, 36-50, college	1.080	4.403	3.040	0.098	4.530	2.852
		[0.942; 1.3421]	[3.197; 4.265]		[0.087; 0.106]	[3.442; 4.122]	
9	Female, 51-60, no college	1.073	7.524	6.072	0.282	7.353	4.756
		[0.802; 1.3607]	[5.580; 9.417]		[0.275; 0.292]	[6.146; 7.847]	
10	Female, 51-60, college	1.040	6.528	4.727	0.225	5.363	3.170
		[0.663; 1.4159]	[4.538; 9.825]		[0.209; 0.246]	[3.853; 6.203]	
11	Male, 51-60, no college	0.737	7.415	5.622	0.271	7.611	4.765
		[0.723; 0.9123]	[5.105; 7.546]		[0.258; 0.272]	[5.827; 7.074]	
12	Male, 51-60, college	0.938	4.051	3.852	0.157	4.581	3.237
		[0.716; 1.2335]	[2.867; 4.253]		[0.150; 0.170]	[3.460; 4.557]	

Parameter estimates for equation 5.2, OLS and IV. We estimate the parameters separately by k-group. The first column are the point estimates for β_k. The second column shows estimates for the σ_{kg} for the Copenhagen metro area). The third column shows the average σ_{kg} estimate across commuting zones. Bootstrapped 95% confidence intervals in square brackets (Hall, 1992).

Source: Administrative registers, Statistics Denmark.

Table D.6. Labor Supply Elasticities and Markdowns, by k-group

		IV		OLS	
	k-group	Elasticity	Markdown	Elasticity	Markdown
1	Female, 26-35, no college	6.221	0.857	-0.010	-0.010
2	Female, 26-35, college	9.061	0.889	-0.489	-1.144
3	Male, 26-35, no college	6.606	0.858	1.724	0.619
4	Male, 26-35, college	10.747	0.900	1.535	0.591
5	Female, 36-50, no college	5.096	0.824	1.121	0.519
6	Female, 36-50, college	6.141	0.849	0.249	0.197
7	Male, 36-50, no college	4.325	0.800	1.392	0.574
8	Male, 36-50, college	4.100	0.793	0.369	0.265
9	Female, 51-60, no college	8.426	0.871	1.695	0.616
10	Female, 51-60, college	5.755	0.837	0.956	0.479
11	Male, 51-60, no college	4.508	0.788	1.561	0.598
12	Male, 51-60, college	4.070	0.787	0.657	0.388
	Overall	5.790	0.829	1.109	0.331

Estimated labor supply elasticities (eq. 3.1) and markdowns $\left(\text{md}_{kj} = \frac{\mathcal{E}_{kj}}{1+\mathcal{E}_{kj}}\right)$ from the labor supply model. Mean of the pooled (over time) distribution of establishment-level labor supply elasticities and markdowns for each k-group. We estimate the parameters separately by k-group. The first two columns report the IV estimates, the third and fourth columns report the OLS estimates.

Table D.7. Substitution Parameter Estimates Across k-groups

		IV			IV	OLS
	k-group	$\rho_k - 1$	$\delta(\rho_k - 1)$	δ	ρ_k	ρ_k
1	Female, 26-35, no college	0.005	0.005	0.806	1.005	0.985
		[-0.004; 0.012]	[-0.002; 0.010]	[0.804; 0.809]	[0.997; 1.012]	[0.982; 0.988]
2	Female, 26-35, college	0.029	0.028		1.029	0.985
		[0.019; 0.038]	[0.019; 0.037]		[1.019; 1.038]	[0.981; 0.988]
3	Male, 26-35, no college	0.007	0.006		1.007	0.987
		[0.000; 0.014]	[0.000; 0.012]		[1.000; 1.014]	[0.985; 0.989]
4	Male, 26-35, college	0.028	0.029		1.028	0.981
		[0.016; 0.036]	[0.017; 0.037]		[1.016; 1.036]	[0.978; 0.984]
5	Female, 36-50, no college	0.016	0.016		1.016	0.978
		[0.006; 0.026]	[0.007; 0.025]		[1.006; 1.026]	[0.976; 0.980]
6	Female, 36-50, college	0.002	-0.004		1.002	0.992
		[-0.0114; 0.0201]	[-0.018; 0.012]		[0.989; 1.020]	[0.987; 0.996]
7	Male, 36-50, no college	-0.024	-0.022		0.976	0.977
		[-0.033; -0.015]	[-0.030; -0.013]		[0.967; 0.985]	[0.975; 0.979]
8	Male, 36-50, college	-0.065	-0.067		0.935	0.999
		[-0.0832; -0.0505]	[-0.084; -0.053]		[0.917; 0.949]	[0.995; 1.003]
9	Female, 51-60, no college	0.003	0.002		1.003	0.990
		[-0.0094; 0.0159]	[-0.010; 0.013]		[0.991; 1.016]	[0.987; 0.993]
10	Female, 51-60, college	-0.027	-0.034		0.973	1.017
		[-0.0538; 0.0022]	[-0.060; -0.004]		[0.946; 1.002]	[1.009; 1.026]
11	Male, 51-60, no college	-0.016	-0.013		0.984	0.985
		[-0.0276; -0.0053]	[-0.025; -0.003]		[0.972; 0.995]	[0.981; 0.988]
12	Male, 51-60, college	-0.036	-0.041		0.964	1.026
		[-0.053; -0.007]	[-0.058; -0.014]		[0.948; 0.993]	[1.020; 1.035]

Parameter estimates for the production function, IV. The first two columns are the point estimates for $(\rho_k - 1)$ and $\delta(\rho_k - 1)$ from equation 5.6. The third and fourth columns show the implied values for δ and ρ_k. The fifth column shows the OLS estimate for ρ_k. Bootstrapped 95% confidence intervals in square brackets.

Source: Administrative registers, Statistics Denmark.

Table D.8. Distribution of Labor Demand Elasticities η_{kjt}, by k-group.

		η_{kjt}			
	k-group	Mean	Median	P10	P90
1	Female, 26-35, no college	-27.070	-9.859	-70.952	-2.187
2	Female, 26-35, college	21.871	-8.528	-83.959	102.098
3	Male, 26-35, no college	9.423	-5.557	-23.556	-1.887
4	Male, 26-35, college	-60.934	-9.597	-75.196	71.058
5	Female, 36-50, no college	-9.958	-7.228	-37.837	-2.001
6	Female, 36-50, college	-28.406	-12.042	-52.689	-2.990
7	Male, 36-50, no college	-4.003	-2.961	-7.104	-1.488
8	Male, 36-50, college	-4.884	-4.326	-8.573	-2.058
9	Female, 51-60, no college	-24.150	-10.801	-52.521	-2.658
10	Female, 51-60, college	-13.663	-12.035	-27.345	-2.845
11	Male, 51-60, no college	-6.225	-4.537	-12.166	-1.964
12	Male, 51-60, college	-8.461	-7.265	-16.165	-2.640

Moments of the firm-level labor demand elasticities η_{kjt} as defined in Section 3.2, eq. 3.5.

Table D.9. Morishima Elasticity of Substitution Between k-groups.

kgroup (k)		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Female, 26-35, no college	**1**	0	-42	-161	-214	-78	510	53	199	-188	2305	276	259
Female, 26-35, college	**2**	-168	0	-116	-36	-69	-739	-30	25	-138	38	-25	14
Male, 26-35, no college	**3**	-183	-45	0	-38	-62	-471	23	-48	-189	12	32	27
Male, 26-35, college	**4**	135	-35	-123	0	-63	778	37	16	204	673	170	87
Female, 36-50, no college	**5**	-156	-34	-130	-35	0	-446	-14	19	-144	26	-13	24
Female, 36-50, college	**6**	-625	2	-95	-347	20	0	-231	25	-470	3160	239	365
Male, 36-50, no college	**7**	54	-48	-93	8	-88	304	0	17	156	14	60	23
Male, 36-50, college	**8**	192	-27	-80	-3	-59	690	43	0	285	178	97	34
Female, 51-60, no college	**9**	-335	-64	-206	-284	-92	55	199	239	0	2411	313	243
Female, 51-60, college	**10**	727	-32	-290	106	-73	2681	41	-16	594	0	110	17
Male, 51-60, no college	**11**	185	-42	-131	29	-121	430	51	15	173	-129	0	16
Male, 51-60, college	**12**	388	-46	-143	42	-69	1609	41	6	222	107	78	0

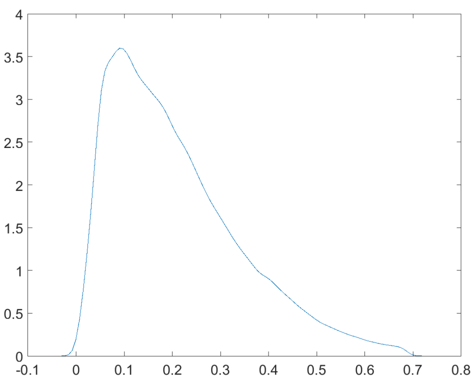
Each cell is the mean Morishima elasticity of substitution calculated across all firms which employ both types of labor.

Table D.10. Variance Decomposition of Counterfactual Wages

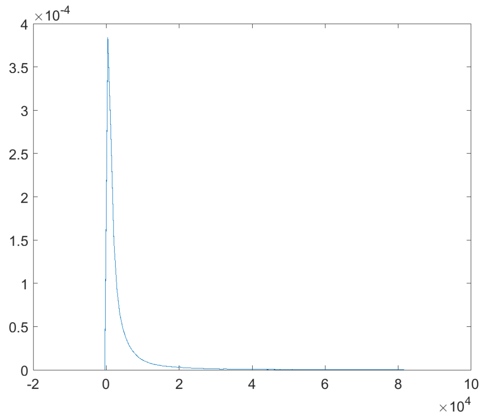
Scenario 1						
Counterfactual Exercise:	Truth	A	B	C	D	E
Variance of Log Wages	0.1285	0.427	0.3346	0.2764	0.2031	0.0005
Variance of Log Markdown	0.0067	0.0067	0.0004	0.0004	0.0003	0.0003
Variance of Log MRPL	0.1394	0.3967	0.3309	0.2723	0.1987	0.0001
2 × Covariance	-0.0176	0.0237	0.0033	0.0037	0.0041	0.0001
Scenario 2						
Counterfactual Exercise:	Truth	C	A	B	D	E
Variance of Log Wages	0.1285	0.086	0.3876	0.2764	0.2031	0.0005
Variance of Log Markdown	0.0067	0.0067	0.0067	0.0004	0.0003	0.0003
Variance of Log MRPL	0.1394	0.08	0.3349	0.2723	0.1987	0.0001
2 × Covariance	-0.0176	-0.0007	0.046	0.0037	0.0041	0.0001
Scenario 3						
Counterfactual Exercise:	Truth	C	D	A	B	E
Variance of Log Wages	0.1285	0.086	0.0912	0.2827	0.2031	0.0005
Variance of Log Markdown	0.0067	0.0067	0.0067	0.0066	0.0003	0.0003
Variance of Log MRPL	0.1394	0.08	0.0942	0.2405	0.1987	0.0001
2 × Covariance	-0.0176	-0.0007	-0.0097	0.0356	0.0041	0.0001
Scenario 4						
Counterfactual Exercise:	Truth	A	B	D	C	E
Variance of Log Wages	0.1285	0.427	0.3346	0.3087	0.2031	0.0005
Variance of Log Markdown	0.0067	0.0067	0.0004	0.0004	0.0003	0.0003
Variance of Log MRPL	0.1394	0.3967	0.3309	0.3048	0.1987	0.0001
2 × Covariance	-0.0176	0.0237	0.0033	0.0035	0.0041	0.0001
Scenario 5						
Counterfactual Exercise:	Truth	D	C	A	B	E
Variance of Log Wages	0.1285	0.1908	0.0912	0.2827	0.2031	0.0005
Variance of Log Markdown	0.0067	0.0067	0.0067	0.0066	0.0003	0.0003
Variance of Log MRPL	0.1394	0.21	0.0942	0.2405	0.1987	0.0001
2 × Covariance	-0.0176	-0.0259	-0.0097	0.0356	0.0041	0.0001
Scenario 6						
Counterfactual Exercise:	Truth	B	A	D	C	E
Variance of Log Wages	0.1285	0.1573	0.3346	0.3087	0.2031	0.0005
Variance of Log Markdown	0.0067	0.0004	0.0004	0.0004	0.0003	0.0003
Variance of Log MRPL	0.1394	0.157	0.3309	0.3048	0.1987	0.0001
2 × Covariance	-0.0176	0	0.0033	0.0035	0.0041	0.0001
Scenario 7						
Counterfactual Exercise:	Truth	D	C	B	A	E
Variance of Log Wages	0.1285	0.1908	0.0912	0.1101	0.2031	0.0005
Variance of Log Markdown	0.0067	0.0067	0.0067	0.0004	0.0003	0.0003
Variance of Log MRPL	0.1394	0.21	0.0942	0.1102	0.1987	0.0001
2 × Covariance	-0.0176	-0.0259	-0.0097	-0.0005	0.0041	0.0001

Counterfactual estimates of the variance of log wages, decomposed into the variances of log markdowns and log MRPL and (2×) the covariance from eq.(5.4), for 7 different decomposition scenarios. In each scenario, each column represents a cumulative counterfactual exercise, where the effect is inclusive of previous columns. For example, Scenario 1 column 3 includes both exercise A and B and Column 4 includes exercises A, B and C. Exercise A sets $u_{jk} = \overline{u}$, B sets $\beta_k = \overline{\beta}$ and $\sigma_{gk} = \overline{\sigma}$, C sets $\gamma_{kj} = \overline{\gamma}$ and $\rho_k = \overline{\rho}$, D sets $\theta_j^{\alpha_j} = \overline{\theta^{\alpha}}$ and $\alpha_j = \overline{\alpha}$, and E sets $\alpha_j = 1$. The overline represents the observation-weighted mean, except in D where it is the median.

Figure D.1. Distribution of Scale (α_{jt}) and Firm Productivity ($\tilde{\theta}_{jt}^{\alpha_{jt}}$).



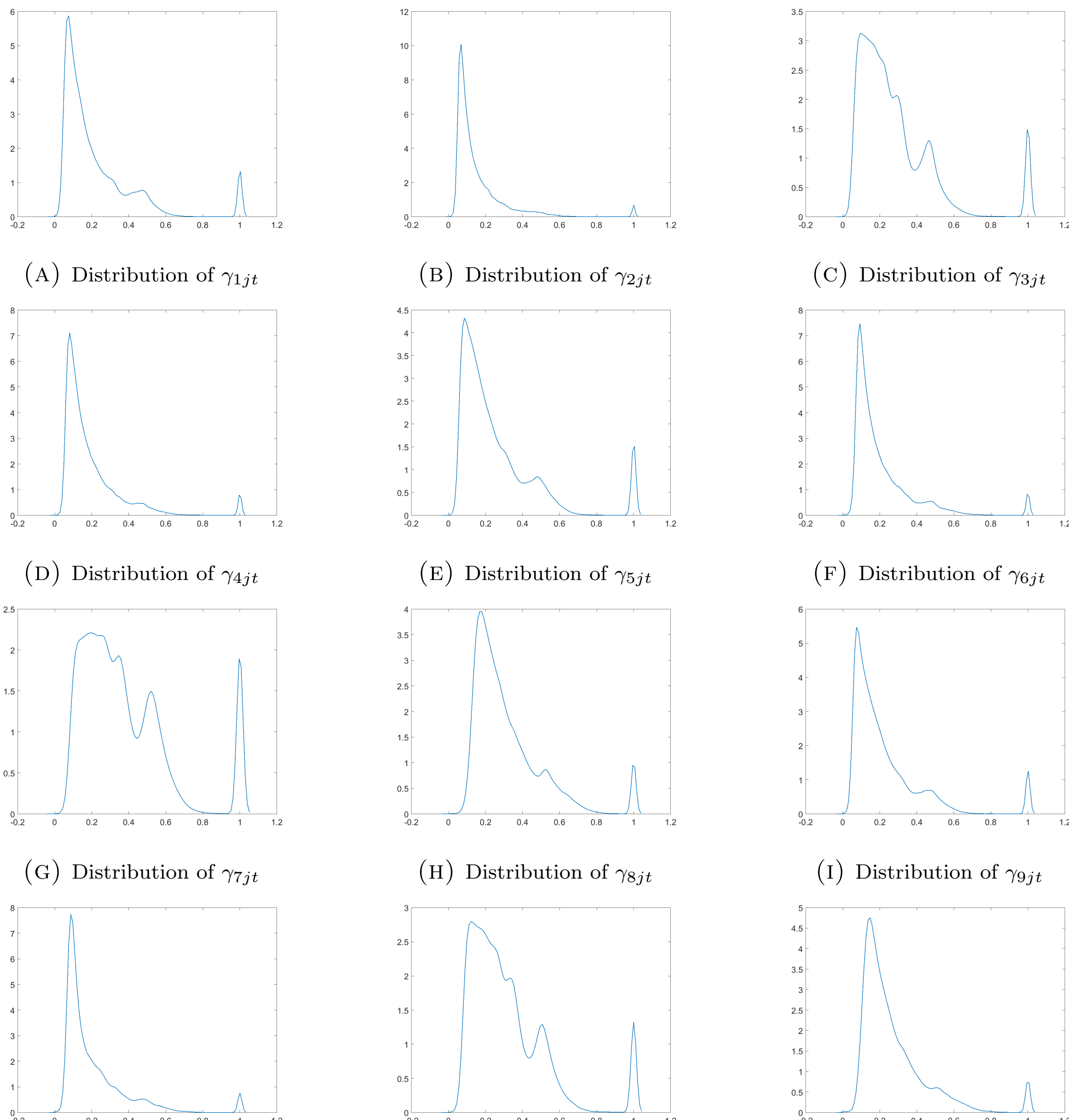
(A) Distribution of α_{jt}



(B) Distribution of $\tilde{\theta}_{jt}^{\alpha_{jt}}$

Panel (a) shows the distribution of the scale parameter α_{jt} (eq. 5.8). The mean of this distribution is 0.214 and the median is 0.181. Panel (b) shows the distribution of productivity term $\tilde{\theta}_{jt}^{\alpha_{jt}}$, truncated at the 99th percentile (eq. 5.9). The mean of the truncated distribution is 6,693 (in 2021 Danish krona). The 90-10 ratio for $\tilde{\theta}_{jt}^{\alpha_{jt}}$ taken over all private sector firms in the economy is 24.3.

FIGURE D.2. Distribution of Normalized Labor Productivity (γ_{kjt}) for each k-group.



(A) Distribution of γ_{1jt} (B) Distribution of γ_{2jt} (C) Distribution of γ_{3jt}

(D) Distribution of γ_{4jt} (E) Distribution of γ_{5jt} (F) Distribution of γ_{6jt}

(G) Distribution of γ_{7jt} (H) Distribution of γ_{8jt} (I) Distribution of γ_{9jt}

(J) Distribution of γ_{10jt} (K) Distribution of γ_{11jt} (L) Distribution of γ_{12jt}

The 12 panels show the distribution of the normalized productivity parameter γ_{kjt} for each of the 12 k-groups (eq. 5.7). The mean and medians of these distributions by k-group are in Table D.8.

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