ONLINE APPENDIX
An Empirical Framework for Matching with Imperfect Competition.

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Appendix C. Nested Logit Economy.

C.1. **Elasticities, Cross-wage super-elasticities, Equilibrium Uniqueness.** To allow unobserved workers preferences ϵ_{ij} to be correlated for certain classes of firms, we partition the J firms into G nests, where each nest is a local labor market. The g^{th} nest contains N_g firms. We assume the ϵ_{ij} to be correlated within nests, i.e., $1/\sigma_{kg} = \sqrt{1 - corr(\epsilon_{ij}, \epsilon_{il})}$ for $j \neq l$ where for $(j,l) \in N_g$, and with $\sigma_{kg} \in [1, \infty)$. Despite the nesting structure, we allow each firm to compete with every other firm in the economy, regardless of whether firms belong to the same nest or not.

In this Nested Logit Economy, the social surplus function is given by

$$G_{k\cdot}(v_{k\cdot}) = \ln \Big\{ \overbrace{e^{v_{k0}} + \sum_{g=1}^{G} \Big(\underbrace{\sum_{j \in N_g} e^{v_{kj}\sigma_{kg}}}_{\mathcal{I}_{k,g}(v_{k\cdot})} \Big)^{1/\sigma_{kg}}}^{\mathcal{I}_{k,M}(v_{k\cdot})} \Big\},$$

where $\mathcal{I}_{k,g}(v_{k\cdot})$ and $\mathcal{I}_{k,M}(v_{k\cdot})$ denote, respectively, the aggregate weighted wage index at the local market g level, and at the "national" market level. Additionally, the market shares have the following weakly separable functional form: $s_{kj}(w_{k\cdot}) = f\big(w_{kj}, \mathcal{I}_{k,g}(v_{k\cdot}), \mathcal{I}_{k,M}(v_{k\cdot})\big)$. The labor supply elasticities are given by:

$$\begin{aligned}\mathcal{E}_{kj} = \frac{w_{kj}}{s_{kj}} \Bigg[& f_1\big(w_{kj}, \mathcal{I}_{k,g}(v_{k\cdot}), \mathcal{I}_{k,M}(v_{k\cdot})\big) + \frac{\partial \mathcal{I}_{k,g}(v_{k\cdot})}{\partial w_{kj}} f_2\big(w_{kj}, \mathcal{I}_{k,g}(v_{k\cdot}), \mathcal{I}_{k,M}(v_{k\cdot})\big) \\ & + \frac{\partial \mathcal{I}_{k,M}(v_{k\cdot})}{\partial w_{kj}} f_3\big(w_{kj}, \mathcal{I}_{k,g}(v_{k\cdot}), \mathcal{I}_{k,M}(v_{k\cdot})\big) \Bigg]\end{aligned}$$

where $f_k(x_1, x_2, x_3) = \frac{\partial f(x_1,x_2,x_3)}{\partial x_k}$ for $k \in \{1, 2, 3\}$. The last equality shows that a change in w_{kj} has a direct effect on the share s_{kj} captured by $f_1(.)$ and two indirect effects mediated

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by the impact of the change of w_{kj} on the local and the total market indexes $\mathcal{I}_{k,g}(v_{k\cdot})$, and $\mathcal{I}_{k,M}(v_{k\cdot})$, respectively.

The elasticity of labor supply in the Nested Logit economy takes the following form:

$$\mathcal{E}_{kj} \;\; = \;\; \beta_{kj}[\sigma_{kg} + (1-\sigma_{kg})s_{kj|g} - s_{kj}] \quad \text{for } j \in N_g \tag{C.1}$$

with $s_{kj} \equiv e^{v_{kj}\sigma_{kg}}\mathcal{I}_{k,g}(v_{k\cdot})^{1/\sigma_{kg}-1}\mathcal{I}_{k,M}(v_{k\cdot})^{-1}$, $s_{kg} = \sum_{j\in N_g} s_{kj} = \mathcal{I}_{k,g}(v_{k\cdot})^{1/\sigma_{kg}}\mathcal{I}_{k,M}(v_{k\cdot})^{-1}$, and $s_{kj|g} = \frac{s_{kj}}{s_{kg}} = e^{v_{kj}\sigma_{kg}}\mathcal{I}_{k,g}(v_{k\cdot})^{-1}$ where $s_{kj|g}$ denotes the share of workers of type k working in the firm j as a fraction of the total nest share.

The cross-wage super-elasticities in the Nested Logit model take the following form:

$$\zeta_{kjl} = \beta_{kj}\left[(1-\sigma_{kg})s_{kj|g}\frac{\mathcal{E}_{kjl|g}}{\mathcal{E}_{kj}} - s_{kj}\frac{\mathcal{E}_{kjl}}{\mathcal{E}_{kj}}\right] \tag{C.2}$$

where $\mathcal{E}_{kjl|g}$ denotes the within-nest cross-wage elasticities. The super-elasticity simplifies to:[5]

$$\zeta_{kj} = \beta_{kj}\left[\beta_{kj}(1-\sigma_{kg})s_{kj|g}(1-s_{kj|g})/\mathcal{E}_{kj} - s_{kj}\right]. \tag{C.3}$$

A direct application of Theorem 2 leads to the following result:

Corollary 1. *Whenever Assumptions 1, 2, and 3 (ii) hold and workers idiosyncratic utility shocks have a Nested Logit structure, an equilibrium exists and it is unique.*

The proof is immediate by showing that the sign restriction in Assumption 3 (i) holds in the Nested Logit Economy.

We can compare our framework to existing literature using this Nested Logit Economy. On one hand, Card et al. (2018) and Lamadon, Mogstad and Setzler (2022) consider a special case of imperfect competition which implies that the two indirect effects of changes in w_{kj} are null, i.e., $\frac{\partial \mathcal{I}_{k,g}(v_{k\cdot})}{\partial w_{kj}}f_2(.) + \frac{\partial \mathcal{I}_{k,M}(v_{k\cdot})}{\partial w_{kj}}f_3(.) = 0$. Such an assumption can considerably limit the effect of market power for some firms and impose important restrictions on the nature of strategic interactions. For instance, these frameworks assume away the possibility that some firms are dominant in a certain local market g, in such a way that they may hire a non-negligible share of some types of workers in their local market. Under this assumption, productivity or amenities shocks in firm j that affect w_{kj} do not have any spillover effects onto the equilibrium wage in a different firm j', $w_{kj'}$. Moreover, the atomistic firm assumption implies that $(1-\sigma_{kg})s_{kj|g} - s_{kj} = 0$ for all $(k,j) \in \mathcal{K}\times\mathcal{J}$, and $g \in \{1,...,G\}$. With $\sigma_{kg} > 1$, this implies that $s_{kj|g} = s_{kj} = 0$. Therefore, if we observe in the data that some firms have a significant share of type-k workers in their local market, i.e., $s_{kj|g} > \underline{s}$ for $\underline{s} > 0$, we can reject the atomistic firm assumption. Finally, we always have $[(1-\sigma_{kg})s_{kj|g} - s_{kj}] \leq 0$, which

[5] We could write also the elasticity as a function of the super-elasticity as in Edmond, Midrigan and Xu (2023), i.e., $\mathcal{E}_{kj} = \frac{\zeta_{kj}+\beta_{kj}s_{kj}}{\beta_{kj}^2(1-\sigma_{kg})s_{kj|g}(1-s_{kj|g})}$.

implies that the atomistic firm assumption leads to an overestimation of firms' labor supply elasticities—and thus the markdowns—and cross-wage super-elasticities.

On the other hand, Berger, Herkenhoff and Mongey (2022), impose the weaker condition that $\frac{\partial \mathcal{I}_{k,M}(v_{k\cdot})}{\partial w_{kj}} f_3(.) = 0$; in other words, they allow some firms to be dominant in their local market but no firm has enough power to hire a significant share of some type of workers at the aggregate market level.[6] Their restriction imposes that $s_{kj} = 0$ for all (k, j), but allows $(1 - \sigma_{kg})s_{kj|g} \neq 0$ for some (k, j). Therefore, they also tend to overestimate labor supply elasticities and cross-wage super-elasticities and thus the true markdowns but with a lower bias than the one estimated under the atomistic firm assumption.[7]

C.2. **Comparative statics: Passthrough.** To clarify how our comparative statics results generalize the special cases analyzed in the literature, we consider the Nested Logit Economy. In this case, the lower bound of Proposition 3(ii)-b simplifies to:

$$\left\{ \underbrace{\underbrace{1 - \frac{\beta_{kj}\sigma_{kg}}{\eta_{kj}}}_{LMS} - \beta_{kj}(1-\sigma_{kg})s_{kj|g}\left[\frac{1}{\eta_{kj}} + \beta_{kj}(1-s_{kj|g})\frac{(1-\text{md}_{kj})^2}{\text{md}_{kj}}\right]}_{BHM} + \beta_{kj}s_{kj}\left[\frac{1}{\eta_{kj}} + (1-\text{md}_{kj})\right] \right\}^{-1} \tag{C.4}$$

LMS denotes the passthrough formula obtained in Lamadon, Mogstad and Setzler (2022) where firms are atomistic, i.e., $s_{kj|g} = s_{kj} \approx 0$. BHM represents the passthrough formula in the Berger, Herkenhoff and Mongey (2022) framework where strategic interactions channels are shut down, i.e., only one dominant firm per local market.[8] Here, our lower bound provides the general formula for the passthrough when all cross-wage elasticities and cross-wage super-elasticities are assumed to be zero, i.e., $\mathcal{E}_{kjl} = \zeta_{kjl} = 0$ for $l \neq j$, i.e., shutting down all strategic interaction channels. No specific restrictions are imposed on $\mathcal{E}_{kj}$ and ζ_{kj}.

[6] In their context, this restriction arises as they consider a model with an infinite number of local markets.

[7] When firms compete according to Bertrand, the labor supply elasticity in Berger, Herkenhoff and Mongey (2022) is given by: $\mathcal{E}_{kj} = [\theta s_{kj|g} + \eta(1 - s_{kj|g})]$ which is a special case of our elasticity when $\theta = \beta_{kj}$, $\eta = \beta_{kj}\sigma_{kg}$ and $s_{kj} = 0$.

[8] In the Berger, Herkenhoff and Mongey (2022) case, the markdown is restricted to the case where $s_{kj} = 0$.

Appendix D. Additional derivations and results.

D.1. **Comparative Statics.** We exploit special features of the Jacobian of our model equilibrium to study comparative statics for the effect on equilibrium wages of changes in total factor productivity (TFP), amenities, and non-employment benefit shocks. We derive closed-form comparative statics for the duopsony version of our model and lower bounds for the general oligopsony version.

Recall the shorthand notation for the derivative of the log wage of type-k workers at firm j with respect to log wages of type-k workers at firm l:

$$\psi_{k,jl} = \frac{\partial \ln \mathrm{mpl}_{kj}}{\partial \ln w_{kl}} + \frac{\partial \ln \mathrm{md}_{kj}}{\partial \ln w_{kl}} \equiv \frac{\mathcal{E}_{kjl}}{\eta_{kj}} + (1 - \mathrm{md}_{kj})\zeta_{kjl}$$

$$\phi_{k,jl} = \frac{\partial \ln \mathrm{mpl}_{kj}}{\partial \ln u_{kl}} + \frac{\partial \ln \mathrm{md}_{kj}}{\partial \ln u_{kl}}.$$

Here is the complete version of Proposition 2 from the main text:

Proposition 3 (Comparative Statics)**.** *Consider that Assumptions 1, 2, and 3 hold. Let (s, w) denote the unique equilibrium outcome of our many-to-one matching model. In a neighborhood of the equilibrium (s, w) the following (general equilibrium) comparative statics hold:*

(i) **Duopsony***: $\mathcal{J} = \{j, l\}$. For any $k \in \mathcal{C}^j \cap \mathcal{C}^l$, we have*

(a)

$$\frac{w_{k0}}{w_{kj}}\frac{\partial w_{kj}}{\partial w_{k0}} = \frac{(1-\psi_{k,ll})\psi_{k,j0} + \psi_{k,jl}\psi_{k,l0}}{(1-\psi_{k,jj})(1-\psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \geq 0.$$

(b) If the firms' production functions have a multiplicative structure of the form $F^l(.) = \check{\theta}_l \check{F}^l(.)$ where $\frac{\partial \check{F}^l(.)}{\partial \check{\theta}_l} = 0$, then for any $k \in \mathcal{C}^j \cap \mathcal{C}^l$, we have

$$\frac{\check{\theta}_l}{w_{kj}}\frac{\partial w_{kj}}{\partial \check{\theta}_l} = \frac{\psi_{k,jl}}{(1-\psi_{k,jj})(1-\psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \geq 0,$$

$$\frac{\check{\theta}_l}{w_{kl}}\frac{\partial w_{kl}}{\partial \check{\theta}_l} = \frac{(1-\psi_{k,jj})}{(1-\psi_{k,jj})(1-\psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} > 0.$$

(c)

$$\frac{u_{kl}}{w_{kj}}\frac{\partial w_{kj}}{\partial u_{kl}} = \frac{(1-\psi_{k,ll})\phi_{k,jl} + \psi_{k,jl}\phi_{k,ll}}{(1-\psi_{k,jj})(1-\psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \gtreqless 0,$$

$$\frac{u_{kl}}{w_{kl}}\frac{\partial w_{kl}}{\partial u_{kl}} = \frac{(1-\psi_{k,jj})\phi_{k,ll} + \psi_{k,lj}\phi_{k,jl}}{(1-\psi_{k,jj})(1-\psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \gtreqless 0.$$

(ii) **Oligopsony***: $J \geq 2$. For any $k \in \mathcal{C}^j \cap \mathcal{C}^l$, and $l, j \in \mathcal{J}$, we have*

(a) For any $k \in \mathcal{C}^j$, we have:

$$\frac{w_{k0}}{w_{kj}} \frac{\partial w_{kj}}{\partial w_{k0}} \geq \frac{\mathcal{E}_{kj0}/\eta_{kj} + (1 - md_{kj})\zeta_{kj0}}{1 - \mathcal{E}_{kj}/\eta_{kj} - (1 - md_{kj})\zeta_{kj}} \geq 0.$$

(b) If the firms' production functions have a multiplicative structure of the form $F^l(.) = \check{\theta}_l \check{F}^l(.)$ where $\frac{\partial \check{F}^l(.)}{\partial \check{\theta}_l} = 0$, then for any $k \in \mathcal{C}^j \cap \mathcal{C}^l$, we have:

$$\frac{\check{\theta}_l}{w_{kj}} \frac{\partial w_{kj}}{\partial \check{\theta}_l} \begin{cases} \geq \frac{\mathcal{E}_{kjl}/\eta_{kj} + (1-md_{kj})\zeta_{kjl}}{\left(1-\mathcal{E}_{kj}/\eta_{kj} - (1-md_{kj})\zeta_{kj}\right)(1-\mathcal{E}_{kl}/\eta_{kl} - (1-md_{kl})\zeta_{kl})} \geq 0 \text{ if } j \neq l, \\ \geq \frac{1}{(1-\mathcal{E}_{kl}/\eta_{kl} - (1-md_{kl})\zeta_{kl})} > 0, \text{ if } j = l. \end{cases}$$

where $\psi_{k,jl}, \phi_{k,jl} \geq 0$ for $l \neq j$, and $\psi_{k,ll}, \phi_{k,ll} \leq 0$.

Before detailing the proof of Proposition 3, we discuss the intuition behind the comparative statics for non-employment benefit and amenities shocks.

Non-employment benefit shocks. Proposition 3(i)/(ii)-a shows the effect of an exogenous increase of non-employment benefits on the equilibrium wages. The equation in (i)-a shows explicitly the different channels by which an exogenous shock to non-employment benefits affects the equilibrium wages in the duopsony case: An increase of w_{k0} has a direct effect on mpl_{kj} and md_{kj}, and firm j increases w_{kj} in response. An indirect effect is transmitted through firm l: The increase of w_{k0} has a direct effect also on mpl_{kl} and md_{kl}, and firm l increases w_{kl}. This change in w_{kl} affects firm j through $\psi_{k,jl}$ and firm j responds by increasing w_{kj}. This in turn generates a response of firm l through $\psi_{k,lj}$, and so on. This succession of responses converges and leads to a final total increase of equilibrium wages. In sum, the strategic responses are mediated by $\psi_{k,jl}$ and $\psi_{k,lj}$ in the duopsony context.

In the more general case with $J \geq 2$, the strategic interactions are captured by $\psi_{k,jr}$ and $\psi_{k,rj}$ for all $r \in \mathcal{J} \setminus \{j\}$. Proposition 3(ii)-a shows that the indirect effects due to strategic interactions can only amplify the magnitudes of the effect of an exogenous increase of non-employment benefits on the equilibrium wages. Indeed, the lower bound derived in (ii)-a is achieved when there are no strategic interactions, i.e., $\psi_{k,jr} = \psi_{k,rj} = 0$ for all $r \in \mathcal{J} \setminus \{j\}$, which happens for example under the "atomistic" firms assumption imposed in Card et al. (2018) and Lamadon, Mogstad and Setzler (2022) or in the Berger, Herkenhoff and Mongey (2022) framework where each local market contains only one firm.

Amenities shocks. In Proposition 3(i)-c we show the effect of a positive increase of type-k worker preference for firm l amenities on equilibrium wages. The duopsony case shows that in the case of an amenities shock, the indirect effect due to strategic interactions works against the direct effect and does not allow us to determine the sign of the equilibrium effect. An increase of u_{kl} directly affects mpl_{kl} and md_{kl} through $\phi_{k,ll}$ and causes firm l to lower the wage w_{kl}. At the same time, the increase in u_{kl} directly affects mpl_{kj} and md_{kj} through $\phi_{k,jl}$, leading firm j to increase the wage w_{kj} though a competition effect. The opposite changes in w_{kl} and w_{kj} both firms through $\psi_{k,jl}$ and $\psi_{k,lj}$. After a set of iterative responses we have the final effect on equilibrium wages and the net sign of this effect is ambiguous. When the strategic interaction terms are 0, i.e., $\psi_{k,jl} = \psi_{k,lj} = 0$, we have $\frac{u_{kl}}{w_{kl}} \frac{\partial w_{kl}}{\partial u_{kl}} < 0$. But when $\psi_{k,jl}$ and $\psi_{k,lj}$ are not null, the resulting aggregate effect could be positive.

D.1.1. *Proof of Proposition 3.* Under Assumptions 1, 2, and 3, we proved that we have an unique equilibrium w^{eq} such that $w^{eq} = B(w^{eq})$. For sake of simplicity let us ignore the upper-script eq in the rest of the proof. By the Implicit Function Theorem we have:

$$\frac{dw}{dw_{k0}} = \mathbb{J}_\delta^{-1}(w)\frac{\partial B(w)}{\partial w_{k0}},$$
$$\frac{dw}{d\gamma_{kl}} = \mathbb{J}_\delta^{-1}(w)\frac{\partial B(w)}{\partial \gamma_{kl}},$$
$$\frac{dw}{d\theta_l} = \mathbb{J}_\delta^{-1}(w)\frac{\partial B(w)}{\partial \theta_l}.$$

Under Assumption 3, $\mathbb{J}_\delta(w)$ is a block diagonal matrix, more precisely it can be written

$$\underset{(KJ\times KJ)}{\mathbb{J}_\delta(w)} = \begin{pmatrix} \mathbb{J}_{\delta,1\cdot}(w) & 0 & \cdots & 0 \\ 0 & \mathbb{J}_{\delta,2\cdot}(w) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbb{J}_{\delta,K\cdot}(w) \end{pmatrix} \text{ where } \underset{(J\times J)}{\mathbb{J}_{\delta,k\cdot}(w)} = \begin{pmatrix} \frac{\partial \delta_{k1}}{\partial w_{k1}} & \cdots & \frac{\partial \delta_{k1}}{\partial w_{kJ}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \delta_{kJ}}{\partial w_{k1}} & \cdots & \frac{\partial \delta_{kJ}}{\partial w_{kJ}} \end{pmatrix}.$$

Case 1 of the proof of Theorem 2 shows that each $\mathbb{J}_{\delta,k\cdot}(w)$ for $k \in \mathcal{K}$ is positive diagonally dominant, therefore its inverse exists and then we have

$$\underset{(KJ\times KJ)}{\mathbb{J}_\delta^{-1}(w)} = \begin{pmatrix} \mathbb{J}_{\delta,1\cdot}^{-1}(w) & 0 & \cdots & 0 \\ 0 & \mathbb{J}_{\delta,2\cdot}^{-1}(w) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbb{J}_{\delta,K\cdot}^{-1}(w) \end{pmatrix}.$$

We then have $\frac{dw_{m\cdot}}{dw_{k0}} = \mathbb{J}_{\delta,m\cdot}^{-1}(w)\frac{\partial B_{m\cdot}(w)}{\partial w_{k0}}$ where $w_{m\cdot} = \begin{pmatrix} w_{m1} \\ \vdots \\ w_{mJ} \end{pmatrix}$, and $B_{m\cdot}(w) = \begin{pmatrix} B_{m1}(w) \\ \vdots \\ B_{mJ}(w) \end{pmatrix}$.

Our derived bounds come from the linear algebra results on M-matrices and inverse M-matrices, i.e., Carlson and Markham (1979); Fiedler and Pták (1962). In fact, case 1 of the Proof of Theorem 2, shows that any $\mathbb{J}_{\delta,k\cdot}(w)$ for $k \in \mathcal{K}$ is positive diagonally dominant and have non-positive off diagonal elements. Then, $\mathbb{J}_{\delta,k\cdot}(w)$, and $\mathbb{J}_\delta(w)$ are M Matrices. Our proofs widely use the result (4.2) of Fiedler and Pták (1962), which states that if A and B are two M matrices such that $A \leqq B$, then $A^{-1} \geqq B^{-1} \geqq 0$. Let's denote by DA the diagonal matrix formed by the diagonal elements of the matrix A. Under Assumption 3, we have $\mathbb{J}_{\delta,k\cdot}(w) \leq D\mathbb{J}_{\delta,k\cdot}(w) \Rightarrow \mathbb{J}^{-1}_{\delta,k\cdot}(w) \geq [D\mathbb{J}_{\delta,k\cdot}(w)]^{-1} \Rightarrow \mathbb{J}^{-1}_{\delta,k\cdot}(w)\frac{\partial B_{k\cdot}(w)}{\partial w_{k0}} \geq [D\mathbb{J}_{\delta,k\cdot}(w)]^{-1}\frac{\partial B_{k\cdot}(w)}{\partial w_{k0}}$ where the last inequality holds since $\frac{\partial B_{kj}(w)}{\partial w_{k0}} \geq 0$ under Assumption 3.

It follows from the latter inequality that:

$$\frac{\partial w_{kj}}{\partial w_{k0}} \geq \frac{w_{kj}}{w_{k0}}\frac{\psi_{k,j0}}{1-\psi_{k,jj}} \geq 0$$

where $\psi_{k,jl} = \left(\frac{w_{kl}}{\ell_{kj}}\frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kl}}\left(\frac{F^j_{kk}}{F^j_k}\ell_{kj}\right) + \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))}\frac{w_{kl}}{\mathcal{E}_{kj}(w_{k\cdot})}\frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial w_{kl}}\right)$.
This latter inequality becomes evident as soon as you remark that:

$$\frac{\partial \delta_{kj}}{\partial w_{kl}}\begin{cases} -\left(\frac{w_{kj}}{w_{kl}}\right)\psi_{k,jl} \text{ if } j \neq l \\ 1-\psi_{k,jl} \text{ if } j = l \end{cases}$$

This proves the first set of bounds.

Second, for $a_{ll} > 0$ and $a_{jl} \leq 0$ when $j \neq l$ it can be shown that

$$H^{-1}(a_{\cdot\cdot}) \equiv \begin{pmatrix} a_{11} & 0 & \cdots & 0 & a_{1l} & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & 0 & a_{ll} & 0 & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & a_{l+1,l+1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & a_{J,J} \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 1/a_{11} & 0 & \cdots & 0 & -a_{1l}/a_{11}a_{ll} & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & 0 & 1/a_{ll} & 0 & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & 1/a_{l+1,l+1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 1/a_{J,J} \end{pmatrix},$$

$$\frac{\partial B_{k\cdot}(w)}{\partial \theta_l} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ B_{kl}(w)/\theta_l \\ 0 \\ \vdots \\ 0 \end{pmatrix} \geq 0.$$

For $a_{jl} \equiv \frac{\partial \delta_{kj}}{\partial w_{kl}}$, we have

$$\mathbb{J}_{\delta,k\cdot}(w) \leq H\left(\frac{\partial \delta_{k\cdot}}{\partial w_{k\cdot}}\right) \Rightarrow \mathbb{J}^{-1}_{\delta,k\cdot}(w) \geq \left[H\left(\frac{\partial \delta_{k\cdot}}{\partial w_{k\cdot}}\right)\right]^{-1} \Rightarrow \mathbb{J}^{-1}_{\delta,k\cdot}(w)\frac{\partial B_{k\cdot}(w)}{\partial \theta_l} \geq \left[H\left(\frac{\partial \delta_{k\cdot}}{\partial w_{k\cdot}}\right)\right]^{-1}\frac{\partial B_{k\cdot}(w)}{\partial \theta_l}.$$

The latter inequality implies that for $j \leq l$ we have:

$$\frac{\partial w_{kj}}{\partial \theta_l}\begin{cases} \geq -\frac{\frac{\partial \delta_{kj}}{\partial w_{kl}}}{\frac{\partial \delta_{kj}}{\partial w_{kj}}\frac{\partial \delta_{kl}}{\partial w_{kl}}}\frac{B_{kl}(w)}{\theta_l} = \frac{w_{kj}\psi_{k,jl}}{\theta_l(1-\psi_{k,jj})(1-\psi_{k,ll})} \geq 0 \text{ if } j<l \\ \geq \frac{1}{\frac{\partial \delta_{kl}}{\partial w_{kl}}}\frac{B_{kl}(w)}{\theta_l} = \frac{w_{kl}}{\theta_l(1-\psi_{k,ll})} > 0, \text{ if } j=l. \text{ otherwise.}\end{cases} \tag{D.1}$$

For $j < l$, we can follow the same process by considering H as a lower triangular matrix. The exact same proof holds for $\frac{\partial w_{kj}}{\partial \theta_l}$. This completes the proof.

Special case: Duopsony. In this special case, we could have a passthrough formula that will hold at equality. This will allow us to have an intuition of the shock transmission from a firm j to a firm l. Recall that $\frac{dw_{m\cdot}}{dw_{k0}} = \mathbb{J}^{-1}_{\delta,m\cdot}(w)\frac{\partial B_{m\cdot}(w)}{\partial w_{k0}}$, and $\frac{\partial \delta_{kj}}{\partial w_{kl}} = -\left(\frac{w_{kj}}{w_{kl}}\right)\psi_{k,jl}$ for $l \neq j$.

Now, consider that $\mathcal{J} = \{j, l\}$. In this special case the inverse of the Jacobian matrix is given by:

$$(\mathbb{J}_{\delta,k\cdot}(w))^{-1} = \begin{pmatrix}\frac{\partial \delta_{kj}}{\partial w_{kj}} & \frac{\partial \delta_{kj}}{\partial w_{kl}} \\ \frac{\partial \delta_{kl}}{\partial w_{kj}} & \frac{\partial \delta_{kl}}{\partial w_{kl}}\end{pmatrix}^{-1} = \frac{1}{(1-\psi_{k,jj})(1-\psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}}\begin{pmatrix}(1-\psi_{k,ll}) & \left(\frac{w_{kj}}{w_{kl}}\right)\psi_{k,jl} \\ \left(\frac{w_{kl}}{w_{kj}}\right)\psi_{k,lj} & (1-\psi_{k,jj})\end{pmatrix}.$$

Then, we can easily derive the following:

$$\frac{w_{k0}}{w_{kj}}\frac{\partial w_{kj}}{\partial w_{k0}} = \frac{(1-\psi_{k,ll})\psi_{k,j0} + \psi_{k,jl}\psi_{k,l0}}{(1-\psi_{k,jj})(1-\psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \geq 0 \tag{D.2}$$

$$\frac{u_{kl}}{w_{kj}}\frac{\partial w_{kj}}{\partial u_{kl}} = \frac{(1-\psi_{k,ll})\phi_{k,jl} + \psi_{k,jl}\phi_{k,ll}}{(1-\psi_{k,jj})(1-\psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \gtreqless 0 \tag{D.3}$$

$$\frac{u_{kl}}{w_{kl}}\frac{\partial w_{kl}}{\partial u_{kl}} = \frac{(1-\psi_{k,jj})\phi_{k,ll} + \psi_{k,lj}\phi_{k,jl}}{(1-\psi_{k,jj})(1-\psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \gtreqless 0 \tag{D.4}$$

$$\frac{\theta_l}{w_{kj}}\frac{\partial w_{kj}}{\partial \theta_l} = \frac{\psi_{k,jl}}{(1-\psi_{k,jj})(1-\psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \geq 0 \tag{D.5}$$

$$\frac{\theta_l}{w_{kj}}\frac{\partial w_{kj}}{\partial \theta_l} = \frac{(1-\psi_{k,jj})}{(1-\psi_{k,jj})(1-\psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \geq 0 \tag{D.6}$$

where the signs restrictions hold, because $\psi_{k,jl}, \phi_{k,jl} \geq 0$ for $l \neq j$, and $\psi_{k,ll}, \phi_{k,ll} \leq 0$ with $\phi_{k,jl} = \left(\frac{u_{kl}}{\ell_{kj}}\frac{\partial \ell_{kj}(w_{k\cdot})}{\partial u_{kl}}\left(\frac{F^j_{kk}}{F^j_k}\ell_{kj}\right) + \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))}\frac{u_{kl}}{\mathcal{E}_{kj}(w_{k\cdot})}\frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial u_{kl}}\right)$.

D.2. **Recovering unobserved types.** The proposed identification strategy requires us to observe at least two time periods. We consider the following potential outcomes model:

$$Y_{it} = \sum_{j \in \mathcal{J}_0}[\ln w_{\mathbf{k}jt} + \eta_{ijt}]1\{D_{it} = j\}, \quad t \in \{1, ..., T\} \tag{D.7}$$

where Y_{it} denotes the observed log earnings of individual i at time t, and $1\{\cdot\}$ denotes the indicator function. $Y_{ijt} \equiv \ln w_{\mathbf{k}jt} + \eta_{ijt}$ denotes potential log earnings if individual i was externally assigned to work at firm j in period t. The potential outcomes are decomposed into two parts (i) $\ln w_{\mathbf{k}jt}$ is the log equilibrium wage, and (ii) η_{ijt} is measurement error or an i.i.d. worker-firm match effect realized after potential mobility across periods.

While in the main text we assumed that the worker's type k is observed by both firms and the econometrician, in general, we could allow k to consist of two subgroups of types, i.e., $k \equiv (\bar{k}, \tilde{k})$, where $\bar{k}$ is defined based on the underlying vector of characteristics $\overline{X}$ that are observed both by the econometrician and firms while $\tilde{k}$ is defined based on the set of characteristics $\widetilde{X}$ that are observable only to firms but not to the econometrician.

Let m_{it} denote the mobility variable, more precisely $m_{it} = 1$ iff $D_{it} \neq D_{it+1}$, i.e., $m_{it} = 1\{D_{it} \neq D_{it+1}\}$. Using shorthand notation $\bar{\mathbf{k}}^{\mathbf{t+1}} = (\bar{\mathbf{k}}_{\mathbf{t}}, \bar{\mathbf{k}}_{\mathbf{t+1}})$, consider the following assumption:

Assumption 4 (Time invariance, Mobility, and Serial Dependence)**.** *We impose the following restrictions.*

(i) Time invariance of unobserved types: $\tilde{\mathbf{k}}_{\mathbf{t}} = \tilde{\mathbf{k}}$ *for* $t \in \{1, ..., T\}$.
(ii) Classical errors: $(\eta_{ijt}, \eta_{ilt+1}) \perp (D_{it}, D_{it+1}) | \tilde{\mathbf{k}}, \bar{\mathbf{k}}_{\mathbf{t}}, \bar{\mathbf{k}}_{\mathbf{t+1}}$
(iii) No serial dependence in the errors: $\eta_{ijt} \perp \eta_{ilt+1} | \tilde{\mathbf{k}}, \bar{\mathbf{k}}_{\mathbf{t}}, \bar{\mathbf{k}}_{\mathbf{t+1}}$ *and* $\eta_{ijt} \perp \bar{\mathbf{k}}_{\mathbf{t+1}} | \tilde{\mathbf{k}}, \bar{\mathbf{k}}_{\mathbf{t}}$

Assumption 4(i) requires the unobserved types to be time invariant. In the same spirit as Burdett and Mortensen (1998) and Hagedorn, Law and Manovskii (2017), Assumption 4(ii) requires the errors to not be correlated with sorting and mobility decisions. The intuition is that these errors are realized after the matches between workers and firms have been formed. Assumption 4(iii) requires the measurement errors associated to a specific mover to not be serially dependent.

Under Assumption 4 we can show that

$$\begin{aligned} &\mathbb{P}(Y_{it} \leq y_t, Y_{i,t+1} \leq y_{t+1} | D_{it} = j, D_{it+1} = l, m_{it} = 1, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}) \\ &\quad = \sum_{\tilde{k}} \mathbb{P}_{\tilde{k}j}(y_t | \bar{k}_t) \mathbb{P}^m_{\tilde{k}l}(y_{t+1} | \bar{k}^{t+1}) \mathbb{P}(\tilde{\mathbf{k}} = \tilde{k} | D_{it} = j, D_{it+1} = l, m_{it} = 1, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}) \end{aligned} \quad \text{(D.8)}$$

where

$$\mathbb{P}_{\tilde{k}j}(y_t | \bar{k}_t) \equiv \mathbb{P}(Y_{it} \leq y_t | D_{it} = j, \tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}} = \bar{k}_t), \quad \text{(D.9)}$$

$$\mathbb{P}^m_{\tilde{k}l}(y_{t+1} | \bar{k}^{t+1}) \equiv \mathbb{P}(Y_{i,t+1} \leq y_{t+1} | D_{it+1} = l, m_{it} = 1, \tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}). \quad \text{(D.10)}$$

Whenever the above decomposition holds and the following three requirements hold: (i) Any two firms j and l belong to a connecting cycle as formally defined in Bonhomme, Lamadon and Manresa (2019), Definition 1, (ii) there exists some asymmetry in the worker type composition between different firms, i.e, Bonhomme, Lamadon and Manresa (2019), Assumption 3(i), and (iii) the matrix defined by the joint log earning distribution $\mathbb{P}(Y_{it} \leq y_t, Y_{i,t+1} \leq y_{t+1} | D_{it} = j, D_{it+1} = l, m_{it} = 1, \mathbf{\bar{k}^{t+1}} = \bar{k}^{t+1})$ for different values of (y_t, y_{t+1}) respects a certain rank condition, i.e, Bonhomme, Lamadon and Manresa (2019), Assumption 3(ii). Then Theorem 1 of Bonhomme, Lamadon and Manresa (2019) applies and the following quantities are point identified: $\mathbb{P}_{\tilde{k}j}(y_t|\bar{k}_t)$, $\mathbb{P}^m_{\tilde{k}l}(y_{t+1}|\bar{k}_{t+1})$, and $\mathbb{P}_{jt}(\tilde{k}|\bar{k}_t) \equiv \mathbb{P}(\mathbf{\tilde{k}} = \tilde{k}|D_{it} = j, \mathbf{\bar{k}_t} = \bar{k}_t)$.

These distributions can be parametrically estimated using the EM algorithm entertained in Bonhomme, Lamadon and Manresa (2019). Using this identification result, it is possible to recover equilibrium wages and shares that were initially unobserved to the econometrician. More precisely, we have the following result:

Proposition 4 (Identification of equilibrium wages and shares)**.** *Consider Assumption 4 holds, and the cdf of classical errors* $F_{\eta_{ijt}|\mathbf{k_t}=k_t}(.)$, *and* $F_{\eta_{ilt+1}|\mathbf{k^{t+1}}=k^{t+1}}(.)$ *are known and strictly increasing on* $\mathbb{R}$. *If the following quantities are point identified* $\mathbb{P}_{\tilde{k}j}(y_t|\bar{k}_t)$, $\mathbb{P}^m_{\tilde{k}l}(y_{t+1}|\bar{k}_{t+1})$, $\mathbb{P}_{jt}(\tilde{k}|\bar{k}_t)$*; then we have the following identification result:*

$$w_{kjt} = \exp\left\{y_t - F^{-1}_{\eta_{ijt}|\mathbf{k_t}=k_t}\left(\mathbb{P}_{\tilde{k}j}(y_t|\bar{k}_t)\right)\right\}, \tag{D.11}$$

$$w_{klt+1} = \exp\left\{y_{t+1} - F^{-1}_{\eta_{ilt+1}|\mathbf{k^{t+1}}=k^{t+1}}\left(\mathbb{P}^m_{\tilde{k}l}(y_{t+1}|\bar{k}_{t+1})\right)\right\}, \tag{D.12}$$

$$s_{kjt} = \mathbb{P}_{jt}(\tilde{k}|\bar{k}_t)\frac{s_{\bar{k}jt}}{\sum_{\mathcal{J}_0}\mathbb{P}_{jt}(\tilde{k}|\bar{k}_t)s_{\bar{k}jt}}. \tag{D.13}$$

where $s_{kjt} = \mathbb{P}(D_{it} = j|\mathbf{k_t} = k_t)$ *and* $s_{\bar{k}jt} = \mathbb{P}(D_{it} = j|\mathbf{\bar{k}_t} = \bar{k}_t)$.

Proof of Proposition 4.

$$\begin{aligned}
&\mathbb{P}(Y_{it} \leq y_t, Y_{i,t+1} \leq y_{t+1}|D_{it} = j, D_{it+1} = l, m_{it} = 1, \mathbf{\bar{k}^{t+1}} = \bar{k}^{t+1}) = \\
&= \sum_{\tilde{k}} \mathbb{P}(Y_{it} \leq y_t, Y_{i,t+1} \leq y_{t+1}|D_{it} = j, D_{it+1} = l, \mathbf{\tilde{k}} = \tilde{k}, \mathbf{\bar{k}^{t+1}} = \bar{k}^{t+1}) \\
&\quad \times \underbrace{\mathbb{P}(\mathbf{\tilde{k}} = \tilde{k}|D_{it} = j, D_{it+1} = l, \mathbf{\bar{k}_t} = \bar{k}_t, \mathbf{\bar{k}_{t+1}} = \bar{k}_{t+1})}_{P(\tilde{k}|j,l,\bar{k}^{t+1})}
\end{aligned}$$

$$\begin{aligned}
&= \sum_{\tilde{k}} \mathbb{P}(\ln w_{\mathbf{k}jt} + \eta_{ijt} \leq y_t, \ln w_{\mathbf{k}j,t+1} + \eta_{ilt+1} \leq y_{t+1}|D_{it} = j, D_{it+1} = l, \mathbf{\tilde{k}} = \tilde{k}, \mathbf{\bar{k}^{t+1}} = \bar{k}^{t+1}) \times P(\tilde{k}|j, l, \bar{k}^{t+1}) \\
&= \sum_{\tilde{k}} \mathbb{P}\left(\ln w_{\mathbf{k}jt} + \eta_{ijt} \leq y_t, \ln w_{\mathbf{k}j,t+1} + \eta_{ilt+1} \leq y_{t+1}|\mathbf{\tilde{k}} = \tilde{k}, \mathbf{\bar{k}^{t+1}} = \bar{k}^{t+1}\right) \times P(\tilde{k}|j, l, \bar{k}^{t+1}) \\
&= \sum_{\tilde{k}} \mathbb{P}\left(\ln w_{\mathbf{k}jt} + \eta_{ijt} \leq y_t|\mathbf{\tilde{k}} = \tilde{k}, \mathbf{\bar{k}^{t+1}} = \bar{k}^{t+1}\right) \times \mathbb{P}\left(\ln w_{\mathbf{k}j,t+1} + \eta_{ilt+1} \leq y_{t+1}|\mathbf{\tilde{k}} = \tilde{k}, \mathbf{\bar{k}^{t+1}} = \bar{k}^{t+1}\right) \times P(\tilde{k}|j, l, \bar{k}^{t+1}) \\
&= \sum_{\tilde{k}} \mathbb{P}\left(\ln w_{\mathbf{k}jt} + \eta_{ijt} \leq y_t, \ln w_{\mathbf{k}j,t+1} + \eta_{ilt+1} \leq y_{t+1}|\mathbf{\tilde{k}} = \tilde{k}, \mathbf{\bar{k}^{t+1}} = \bar{k}^{t+1}\right) \times P(\tilde{k}|j, l, \bar{k}^{t+1}) \\
&= \sum_{\tilde{k}} \mathbb{P}\left(Y_{it} \leq y_t|D_{it} = j, \mathbf{\tilde{k}} = \tilde{k}, \mathbf{\bar{k}_t} = \bar{k}_t\right) \times \mathbb{P}\left(Y_{i,t+1} \leq y_{t+1}|D_{it+1} = l, m_{it} = 1, \mathbf{\tilde{k}} = \tilde{k}, \mathbf{\bar{k}^{t+1}} = \bar{k}^{t+1}\right) \times P(\tilde{k}|j, l, \bar{k}^{t+1})
\end{aligned}$$

Now, we have

$$\begin{aligned}
\mathbb{P}_{\tilde{k}j}(y_t|\bar{k}_t) &\equiv \mathbb{P}(Y_{it} \leq y_t|D_{it}=j, \tilde{\mathbf{k}}=\tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t) \\
&= \mathbb{P}(\ln w_{\mathbf{k}jt}+\eta_{ijt} \leq y_t|D_{it}=j, \tilde{\mathbf{k}}=\tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t) \\
&= \mathbb{P}(\ln w_{\mathbf{k}jt}+\eta_{ijt} \leq y_t|\tilde{\mathbf{k}}=\tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t) \\
&= \mathbb{P}(\eta_{ijt} \leq y_t-\ln w_{\mathbf{k}jt}|\tilde{\mathbf{k}}=\tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t) = F_{\eta_{ijt}|\bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t}(y_t-\ln w_{\mathbf{k}jt})
\end{aligned}$$

We can then easily recover the first result by inverting the last equation and obtain:

$$w_{kjt} = \exp\left\{y_t - F^{-1}_{\eta_{ijt}|\bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t}\left(\mathbb{P}_{\tilde{k}j}(y_t|\bar{k}_t)\right)\right\}.$$

The second equality of the proposition could be derived analogously. For the last equality we have:

$$\begin{aligned}
\mathbb{P}(D_{it}=j|\tilde{\mathbf{k}}=\tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t) &= \frac{\mathbb{P}(\tilde{\mathbf{k}}=\tilde{k}|D_{it}=j, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)\times\mathbb{P}(D_{it}=j|\tilde{\mathbf{k}}=\tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)}{\mathbb{P}(\tilde{\mathbf{k}}=\tilde{k}|\bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)} \\
&= \frac{\mathbb{P}(\tilde{\mathbf{k}}=\tilde{k}|D_{it}=j, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)\times\mathbb{P}(D_{it}=j|\tilde{\mathbf{k}}=\tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)}{\sum_j \mathbb{P}(\tilde{\mathbf{k}}=\tilde{k}|D_{it}=j, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)\times\mathbb{P}(D_{it}=j|\tilde{\mathbf{k}}=\tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)}
\end{aligned}$$

□

Parametric estimation and EM algorithm. For practical purposes, we impose a normality distribution for the classical errors, then $\ln w_{kjt}+\eta_{ijt}|\mathbf{k^t}=k^t \sim N\left(\ln w_{kjt}, \varrho_{kjt}\right)$ and $\ln w_{klt}+\eta_{ilt+1}|\mathbf{k^{t+1}}=k^{t+1} \sim N\left(\ln w_{klt+1}, \varrho_{klt+1}\right)$. Let $\tilde{K}$ denote the number of unobserved types, $C_{\bar{k}^t}$ be a set of firms that have been hiring workers of observable types $\bar{k}^t$ over the two periods t and $t+1$ and belonging to a connecting cycle as defined in Bonhomme, Lamadon and Manresa (2019). $N^m_{\bar{k}^t}$ denotes the number of movers with observable types $\bar{k}^t$. First, we consider the following log-likelihood function for job movers:

$$\sum_{i=1}^{N^m_{\bar{k}^t}} \sum_{j\in C_{\bar{k}^t}} \sum_{l\in C_{\bar{k}^t}} \ln\left(\sum_{\tilde{k}=1}^{\tilde{K}} p_{\tilde{k}jl}\frac{1}{\sqrt{4\pi^2\varrho_{(\tilde{k},\bar{k}_t)jt}\varrho_{(\tilde{k},\bar{k}_t)lt+1}}}e^{-\frac{\left(y_{it}-\ln w_{(\tilde{k},\bar{k}_t)jt}\right)^2}{2\varrho^2_{(\tilde{k},\bar{k}_t)jt}}-\frac{\left(y_{it+1}-\ln w_{(\tilde{k},\bar{k}_t)lt+1}\right)^2}{2\varrho^2_{(\tilde{k},\bar{k}_t)lt+1}}}\right) \tag{D.14}$$

where $\hat{w}_{(\tilde{k},\bar{k}_t)jt}$, $\hat{w}_{(\tilde{k},\bar{k}_t)lt+1}$, $\hat{\varrho}_{(\tilde{k},\bar{k}_t)jt}$, $\hat{\varrho}_{(\tilde{k},\bar{k}_t)lt+1}$, and $\hat{p}_{\tilde{k}jl}$ for $\tilde{k}=1,...,\tilde{K}$ are estimated by maximizing (D.15) using the EM algorithm.

Second, we consider the log-likelihood of the for all workers at the period t:

$$\sum_{i=1}^{N_{\bar{k}^t}} \sum_{j\in C_{\bar{k}_t}} \ln\left(\sum_{\tilde{k}=1}^{\tilde{K}} q_{\tilde{k}jt}\frac{1}{\sqrt{4\pi^2\hat{\varrho}_{(\tilde{k},\bar{k}_t)jt}}}e^{-\frac{\left(y_{it}-\ln\hat{w}_{(\tilde{k},\bar{k}_t)jt}\right)^2}{2\hat{\varrho}^2_{(\tilde{k},\bar{k}_t)jt}}}\right) \tag{D.15}$$

where $N_{\bar{k}_t}$ denotes the number of workers with observable types $\bar{k}_t$, and $q_{\tilde{k}jt} \equiv \mathbb{P}_{jt}(\tilde{k}|\bar{k}_t)$. Again we estimate $\hat{q}_{\tilde{k}jt}$ by maximizing equation (D.15) using the EM algorithm. Then we use equation (D.13) to recover $\hat{s}_{kjt}$.

Given employment shares s_{kjt} for each firm and worker type, we can then obtain the total quantity of each worker type in the population, $m_{kt} = \sum_j \ell_{kjt}$, as the (year-by-year) solution to an overdetermined system of linear equations: $S_t m_t = \mu_t$. Here S_t is the known $J \times K$ matrix of worker type shares s_{kjt} at each firm in period t, μ_t is the known $J \times 1$ vector of total employment $\mu_{jt} = \sum_{k \in \mathcal{C}_t^j} \ell_{kjt}$ at each firm, and m_t is the unknown $K \times 1$ vector of individuals m_{kt} of each type k. If both S_t and the associated augmented matrix have rank equal to K, then there will be a unique solution which provides m_{kt} for each period t[9]. We can then obtain $\ell_{kjt} = s_{kjt} m_{kt}$ for each firm, type and year.

Given that we have recovered the equilibrium wages and shares, and number of matches, these objects can then be used to recover the model parameters.

D.3. **Identifying the Labor Supply Parameters.** The baseline labor supply equation from the model is

$$\ln \frac{s_{kjt}}{s_{k0t}} = \overline{u}_k + \beta_{1k} \ln \frac{w_{kjt}}{w_{k0t}} + \sum_{g=1}^{G} \tilde{\sigma}_{kg} \ln s_{kj|gt} \mathbb{1}_{j|g} + \ln u_{kjt} \tag{D.16}$$

where $\tilde{\sigma}_{kg} \equiv (1 - 1/\sigma_{kg})$. Define $\mathbb{1}_{j|g} = 1$ if $j \in g$ and 0 else.

The identification challenge is that both the wage and inside share are potentially correlated with the unobserved amenities and thus endogenous. To address this challenge, we propose and apply in the main text an instrumental variables (IV) strategy which leverages exogenous variation in firm productivity. Here we discuss the application and results from some alternate IV strategies.

One source of instruments relies on strategic interactions between firms in wage-setting. In the presence of strategic interactions, the number and characteristics of other firms in a given labor market can be used as instruments. These so-called "BLP instruments" are very common in the industrial organization literature in the context of the product market where the characteristics and number of competing products are used as instruments for prices (see Berry, Levinsohn and Pakes, 1995 (BLP) for the canonical example). In a labor market context, possible BLP instruments might include the number of firms, average size, or average value-added per worker of other firms in the labor market. Azar, Berry and Marinescu (2022a) use the number of vacancies and log employment of competing firms as instruments for advertised wages on a job posting website. In results not reported, we consider the available BLP instruments in our data, such as the number of firms in the same market, and found that they were not sufficiently strong. Thus, we do not emphasize BLP instruments in our setting.

A second source of wage instruments exploits "uniform wage-setting" whereby firms set wages similarly across local labor markets (Hazell et al., 2022). As suggested by Azar,

[9]This is the Rouché-Capelli theorem.

Berry and Marinescu (2022a), this implies that the wage a firm pays in a given market may be driven by the labor market conditions that same firm faces in other markets. We thus considered Hausman instruments for w_{kjg} in market g using the average predicted wage across all markets that firm operates in other than g[10]. In results not reported, we implemented this approach, following Azar, Berry and Marinescu (2022a), but generally found that these instruments were too weak in our setting.

Finally, we considered a shift-share IV approach following Hummels et al. (2014) and Garin and Silvério (2023) to estimate labor supply. To construct this instrument, we rely on firm-product-country level yearly foreign trade data from Statistics Denmark register UHDI and bilateral trade flows from the BACI dataset. We find that our labor supply parameters are comparable to our main estimates reported in Table F3. We do not emphasize these estimates as much in the paper since we are only able to construct the instrument for the small share of the firms in our sample who export. These results are available upon request.

D.4. **Multi-Equation GMM Approach to Estimating Production Parameters.** Estimating equation (5.8) is not straightforward. We cannot use an equation-by-equation approach as we do for the labor supply equation due to the presence of common parameters across equations. While there are only $K+1$ parameters to estimate ($\rho_k \,\forall\, k$ and δ), there are $K \times (K-1)/2$ equations which could be used to estimate the parameters, with no obvious guidance on which to use. Since not all firms employ every labor type, any subset of equations will somewhat arbitrarily ignore the contribution of some firms. If all firms employed some base type of labor, all the labor ratio equations could be cast in terms of that type. However this is not the case, so an alternative is to use all $K \times (K-1)/2$ equations in a multi-equation GMM estimator. Another possible approach would be to treat the multi-equation GMM system nonlinearly and estimate the $K+1$ parameters directly. This would require $K+1$ instruments, for which the obvious choices are lagged labor and wages for each labor type. However, due to the size of the problem this may be intractable.

The approach we take is to treat the system as a set of linear equations with cross-equation parameter restrictions, estimating the compound parameters—such as $\delta(\rho_k - 1)$—and then calculating the structural parameters post-estimation. This has the advantage of being much faster, and also allows specification testing of the model assumptions—since we can test if our estimates of $\delta(\rho_k - 1)$ equal the product of our estimates of δ and $(\rho_k - 1)$. Functionally, we estimate $K \times (K-1)/2$ equations, where each equation (for all a, b in the set of labor

[10]We also exclude markets in the same municipality or industry as g.

types) takes the following form:

$$d_{kjt}d_{hjt}\log\frac{\tilde{w}_{ajt}}{\tilde{w}_{bjt}} = \sum_k \mathbb{1}_{k=a}d_{kjt}\left[\beta_k^1\log\ell_{kjt} - \beta_k^2\log\ell_{kjt-1}\right] - \sum_h \mathbb{1}_{h=b}d_{hjt}\left[\beta_h^1\log\ell_{hjt} - \beta_h^2\log\ell_{hjt-1}\right] + \sum_{k,h,t}\mathbb{1}_{k=a}\mathbb{1}_{h=b}d_{kjt}d_{hjt}\left[\delta\log\frac{\tilde{w}_{kjt-1}}{\tilde{w}_{hjt-1}} + c_{kht}\right] + \eta_{abjt} \tag{D.17}$$

where $\beta_k^1 \equiv (\rho_k - 1)$, $\beta_k^2 \equiv \delta(\rho_k - 1)$, and d_{kjt} is an indicator variable which equals 1 if firm j employs labor type k in periods t and $t-1$. This is similar to a "multivariate" regression where all the same regressors appear on the RHS of every equation. We now have $2K+1$ parameters to estimate, and thus need $2K+1$ instruments. Here we use lagged labor ℓ_{kjt-1}, lagged wages w_{kjt-1}, plus squares of both, giving us an overidentified system which we estimate using linear GMM (essentially 2SLS). Note that this approach allows for arbitrary cross-equation patterns of correlation between the error terms η_{abjt}.

D.5. **Passthrough of Productivity Shocks.** Following Lamadon, Mogstad and Setzler (2022) and our own estimation strategy, we regress long changes in average establishment-level log wages by k-group over long changes in log firm-level value added per worker ($VAPW_{jt}$), instrumented by short changes in VAPW. Our empirical strategy follows Morelli and Herkenhoff (2025) by interacting the VAPW shock with both the within-market share and the national share. This also extends Berger, Herkenhoff and Mongey (2022) who conduct a similar exercise but only consider market-level oligopoly.

We use the estimation dataset described in Section 6.1 and Online Appendix E. Table D1 column (2), presents the results of the following regression:

$$\Delta_{e,e'}\ln w_{kjt} = \alpha_0 + \alpha_1\Delta_{e,e'}\ln VAPW_{jt} + \alpha_2 s_{kj|gt-3} + \Delta_{e,e'}\ln VAPW_{jt}\times\alpha_3 s_{kj|gt-3} + \alpha_4 s_{kj|t-3} + \alpha_5\Delta_{e,e'}\ln VAPW_{jt}\times s_{kj|t-3}$$

where we set $e = 2$ and $e' = 3$ and the market shares are expressed in percentages. In column (1), we show results of a specification not including market shares. Average k type worker wages go up by 7.2 percent after a 10 percent increase in VAPW.[11] In column (3), we add controls for establishment size, and dummies for firm id, k-group, year, and local labor market. Our estimates indicate that establishments with a relatively larger market share, either local or national, have a relatively lower passthrough rate, consistent with the findings of Morelli and Herkenhoff (2025). This suggests the presence of strategic interactions both at the market and national levels in Denmark.

[11]Running the same specification with establishment-level data rather than establishment-k-group-level data results in a passthrough of 15.6 percent, comparable to the market passthrough estimates for U.S. data from Lamadon, Mogstad and Setzler (2022).

Dependent Variable:	$\Delta_{e,e'} \ln w_{kjt}$ (1)	$\Delta_{e,e'} \ln w_{kjt}$ (2)	$\Delta_{e,e'} \ln w_{kjt}$ (3)
$\Delta_{e,e'} \ln VAPW_{jt}$	0.072***	0.084***	0.067***
	(0.005)	(0.006)	(0.009)
$s_{kj\|gt-3}$		0.000***	0.000***
		(0.000)	(0.000)
$\Delta_{e,e'} \ln VAPW_{jt} \times s_{kj\|gt-3}$		-0.001***	-0.000
		(0.001)	(0.001)
$s_{kj\|t-3}$		0.064***	0.132***
		(0.013)	(0.014)
$\Delta_{e,e'} \ln VAPW_{jt} \times s_{kj\|t-3}$		-0.699***	-0.536***
		(0.145)	(0.126)
Constant	-0.015***	-0.017***	
	(0.000)	(0.000)	
Establishment size	N	N	Y
Firm id FE	N	N	Y
k-group FE	N	N	Y
Year t FE	N	N	Y
g (commuting zone×industry) FE	N	N	Y
Observations	1,093,731	1,093,731	1,093,731

Table D1. Regression of establishment-level long changes in type k average log wages on firm-level long changes in value added per worker on the 3-period lag of the establishment's local labor market share of type-k workers (in percentages) and its interaction with long changes in value added, on the lag of the establishment's national labor market share of type-k workers (in percentages) and its interaction with long changes in value added (2-3). We instrument long changes in log value added with short changes (1-period) in value added per worker. We add controls for the log of establishment size, firm fixed effects, worker type fixed effects, year fixed effects, local labor market fixed effects (3). Robust standard errors in parentheses. Estimation dataset described in Online Appendix E; we drop observations with missing value added data and singleton observations.

Appendix E. Data and Sample Description

Our data consists of several administrative registers provided by Statistics Denmark for the years 2001-2019. These include annual cross-section data from the Danish register-based, matched employer-employee dataset IDA (Integrated Database for Labor Market Research) and other annual datasets, divided into IDAN, IDAS, and IDAP. The datasets are linked by individual identifiers for persons, firms, and establishments. Table E1 lists the relevant datasets and details.

We restrict the dataset to individuals between 26 and 60 years of age who work full-time as employees in the private sector whose job is linked to a physical establishment. We drop individuals employed in the financial sector; firms in the financial sector are not required to report revenue data and very few do. Details on data and sample selection are in Table E2. In total, our dataset consists of $12,742,746$ individual-year combinations. Our sample construction selects the data in a few important ways: The full population of salaried jobs in Denmark in 2001-2019 is 49.3 percent female. This goes down to 35.8 percent when

Category	**Register**	**Variables**
workers	IDAN (jobs yearly panel)	firm and establishment indicator, establishment location, yearly earnings, hours worked, share of the year worked, type of job (primary, secondary), type of job (part-time/full-time), type of job (occupation, DISCO code)
not employed	BEF (population register) IDAN	We classify as not employed all individuals in the relevant age groups who are not recorded in IDAN.
unemployed	IND (income dataset, individual yearly panel), IDAP (worker dataset, individual yearly panel)	unemployment benefits, duration of unemployment
firms and establishments	FIRM, IDAS (workplace panel)	firm revenue and value added, sector of industry (5-digit industry classification based on NACE rev. 2), establishment location (municipality)
k-groups	UDDA (education panel), BEF (individual yearly panel)	age, highest acquired education, gender
commuting zones	Eckert, Hejlesen and Walsh (2022) (available on Fabian Eckert website)	commuting zone (link to municipality)

Table E1. Data Description (Datasets and Variables).

	step	observations	share in public sector	share in financial sector	share full-time	share female	age	avg. yearly earnings (2022 USD)
1	All salaried jobs in Denmark in 2001-2019	76,869,608						
2	Keep jobs held by workers in k-groups	50,263,511	0.229	0.024	0.437	0.493	42.5	42,867
3	Keep jobs with market information	32,486,151	0.355	0.037	0.648	0.487	43	56,389
4	Drop workers in small commuting zones	32,106,644	0.354	0.037	0.768	0.487	43	56,474
5	Drop jobs with no earnings or hours	32,094,227	0.354	0.037	0.648	0.487	43	56,493
6	Drop public sector jobs	20,719,775		0.057	0.660	0.358	42.5	59,641
7	Drop financial sector jobs	19,538,794			0.653	0.349	42.4	58,296
8	Keep full-time, highest-paying jobs	**12,742,741**				0.318	43.5	71,491
9	Keep only period 2004-2016	8,614,259						

Table E2. Worker Sample Selection.

we drop the public sector and further to 31.8 percent when we exclude the financial sector and non-full-time jobs. Workers in the private-sector with full-time jobs are on average one year older than the full worker population, and have average yearly earnings of $71,491$ USD, higher than the full-worker-population average of $42,867$ USD.

Find a detailed description of the selection steps below:

(1) This step excludes self employed and employers, and their spouses if their main source of income is from assisting the spouse's enterprise; it includes all other types of jobs.
(2) This step drops workers not appearing in the population registers, younger and older workers, as well as workers with no education status recorded (this applies mostly to immigrant workers). Therefore, this step excludes jobs held by workers not resident in Denmark.
(3) This step drops jobs without real establishment code, i.e., all non-primary jobs and primary jobs with missing or fictitious establishment code. Primary jobs are the most important connection to the labor market (longest employment period and largest ATP payments). Workers with fictitious workplaces (establishment nr. = 0) are those who cannot be linked to any of the employer's registered workplaces, either because they work from home or in various workplaces (such as cleaners, home nurses). Workers with no workplace (establishment nr. = .) are those with multiple workplaces for which one unique workplace cannot be identified. In 2,491,168 instances, where the establishment information is missing only in one year during a continuous employment spell at the same firm, we impute it.
(4) Drop jobs in establishments in Christiansœ, Bornholm, Samsœ, and Æro.
(5) Drop jobs with no information on earnings or hours
(6) Drop if the sector of industry of the employer is one of the following 1-digit NACE rev.2 codes {O,P,Q,T,U,X}.
(7) Drop if the sector of industry of the employer is nacee-2 code K (this sector has an extreme underreporting of revenue data).
(8) We define full-time jobs as jobs with weekly schedule of 30 hours or more.

We denote establishments with the subscript j, time (years) with the subscript t, and worker type (k-groups) with the subscript k. k-groups are divided by gender (male or female) age group (26-35, 36-50, 51-60) and education level (completed or not tertiary education). We define a local labor market g as a commuting zone and industry pairing. We use the 3-digit EU industry classification NACE Rev. 2 (Carré, 2008) and we drop the public and financial sectors. We use 16 of the 23 commuting zones computed for 2005 by Eckert, Hejlesen and Walsh (2022) using the Tolbert and Sizer (1996) method for Denmark. We drop six of the commuting zones that are small islands relatively separated from the mainland (Christiansœ, Bornholm, Samsœ, and Æro), and we merge the two North Jutland commuting zones of Aalborg and Frederikshavn. In our final estimation dataset, we have 2,757 local labor markets. We collapse the individual-level dataset at the (k, j, t) level leading to $4,487,628$ observations. We restrict the estimation dataset to only establishments with no missing values for any of the key variables. Table E3 details the sample selection process.

	step	total observations	unique establishments
1	collapse at the k-group-establishment-year (k, j, t) level	4,487,620	259,190
2	merge revenue data (firm, year)	-	-
3	add share of non-employed/unemployed and average unemployment income	-	-
4	drop observations with wage bill to revenue ratio above 80 percent (drops all observations with missing revenue)	4,054,229	238,295
	keep observations for firms that appear at least once in the estimation dataset	**3,069,502**	**63,526**
5	create estimation variables	-	-
6	keep observations in 2004-2017 to accommodate for long run lags $(x_{jkt+2} - x_{jkt-3})$ and data break	2,332,058	-
7	drop firms/k-groups with not enough longevity to allow for computing short-run lags $(x_{jkt} - x_{jkt-1})$	2,294,908	-
8	drop firms/k-groups with not enough longevity to allow for computing long-run lags $(x_{jkt+2} - x_{jkt-3})$	1,983,593	-
9	drop firms employing only one k-group (necessary for the second instrument)	**1,101,543**	**63,526**

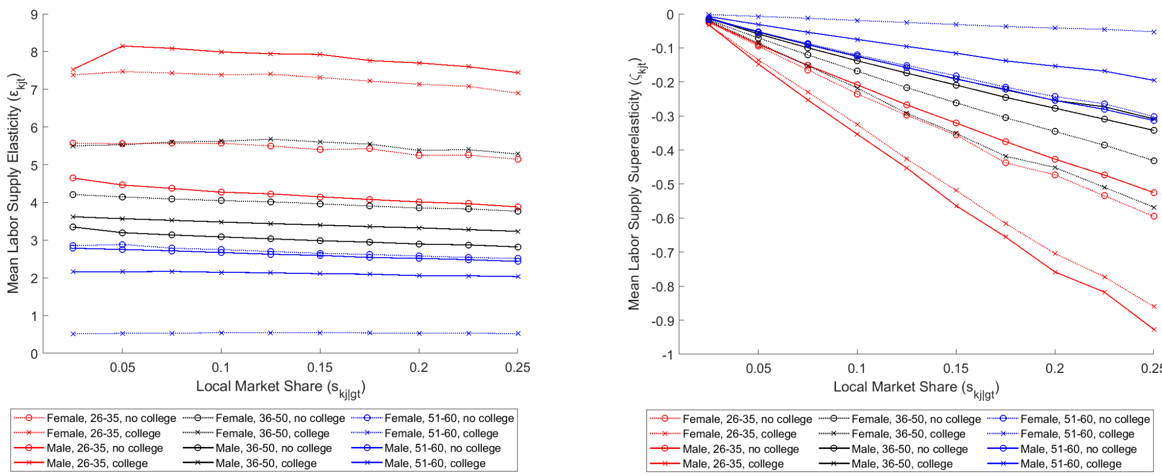
Table E3. Establishment Sample Selection and Construction of the Estimation Dataset. Start with panel of selected workers in years 2001-2019. Variables: full-time-equivalent, earnings, k-group (gender, age, education), local market (commuting zone, industry), firm, establishment, year (12,742,741 individuals).

We measure labor inputs in terms of full-time equivalents (FTE). We calculate the full-time equivalent as the number of hours worked in the calendar year divided by the average number of full-time hours worked by full-time workers in Denmark over the same period, where we define a full-time worker as an individual who works 30+ hours a week. This implies that if an individual works full-time in one establishment for six months, she will be counted as half of a FTE. We use non-employment (unemployment + non-participation) as the outside option. We define non-participation as an individual not observed in the linked employer-employee data for a (part of the) year. Non-participation income is set to zero. Unemployment spells and unemployment income are observed directly in the data. Therefore, non-employment income consists of unemployment income for the unemployed workers. This includes cash assistance, unemployment benefits, leave benefits, and other assistance benefits, but—similarly to our measure of wages—it does not include long-term sickness or pension benefits.

The key variables we use in the estimation are:

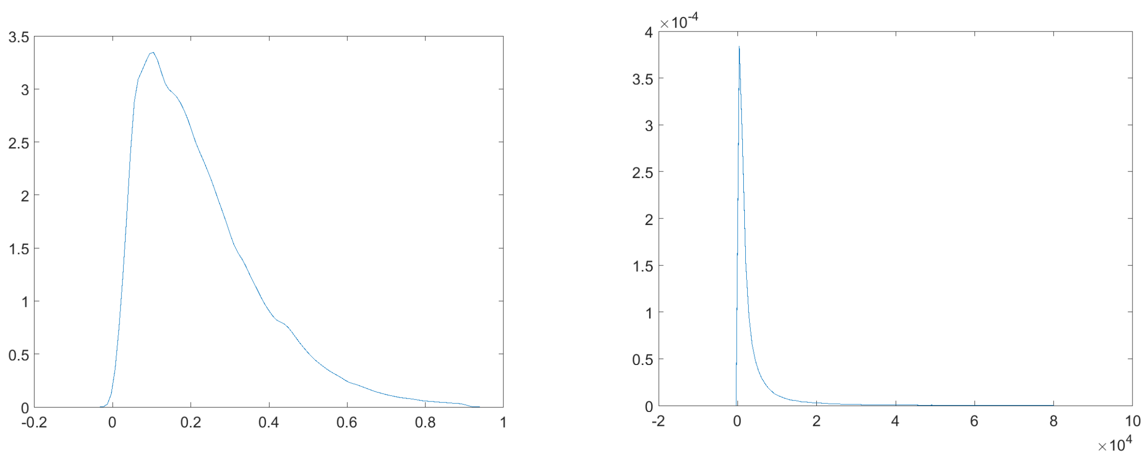
- w_{kjt}: mean earnings by k-group, establishment, year
- w_{k0t}: mean non-employment income by k-group, year
- s_{kjt} and $s_{kj|gt}$: employment shares, by k-group, establishment, year, overall and by market g (inside shares)
- s_{0t}: overall non-employment shares, by k-group, establishment, year (calculated by summing the non-employment spells at the k level and dividing by the total number of FTEs and non-employment spells in the data)
- $s_{\sim kj|gt}$: sum of the inside shares for all other labor types employed by establishment j, by k-group, year, market
- R_{jt}: establishment-level revenue by year, obtained allocating firm revenue across establishments in proportion to their wage bills

Appendix F. Appendix Figures and Tables



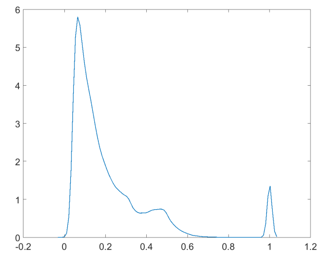
(A) labor supply elasticity ($\mathcal{E}_{kjt}$) (B) cross-wage super-elasticity (ζ_{kjt})

Figure F1. Labor supply elasticities by local market share and worker type. Local market: Commuting Zone×Industry. Panel (a) plots average estimated labor supply elasticities ($\mathcal{E}_{kjt}$) over the local market share ($s_{kj|gt}$), by k-group. Panel (b) plots average estimated labor supply super-elasticities (ζ_{kjt}) over the local market share ($s_{kj|gt}$), by k-group. Establishment-level elasticities are averaged across years and local markets by the establishment local market share bin (10 bins between 0 and 0.25).

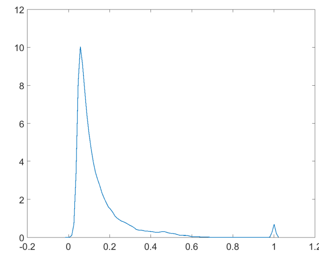


(A) Distribution of α_{jt} (B) Distribution of $\tilde{\theta}_{jt}^{\alpha_{jt}}$

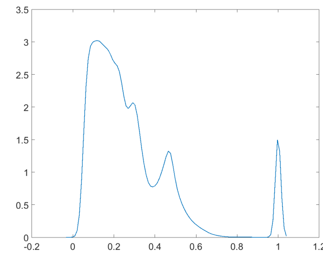
Figure F2. Panel (a) shows the distribution of the scale parameter α_{jt} (equation (5.10)). The mean of this distribution is 0.214 and the median is 0.181. Panel (b) shows the distribution of productivity term $\tilde{\theta}_{jt}^{\alpha_{jt}}$, truncated at the 99th percentile (equation (5.11)). The mean of the truncated distribution is $6,538$ (in 2021 thousands of Danish krona). The 90-10 ratio for $\tilde{\theta}_{jt}^{\alpha_{jt}}$ over all private sector firms in the economy is 22.8.



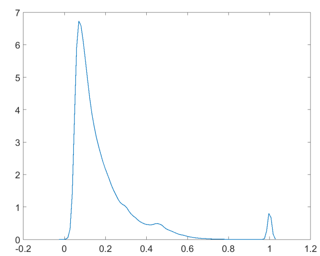
(A) Distribution of γ_{1jt}



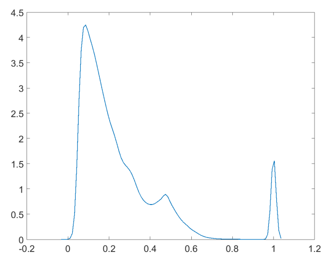
(B) Distribution of γ_{2jt}



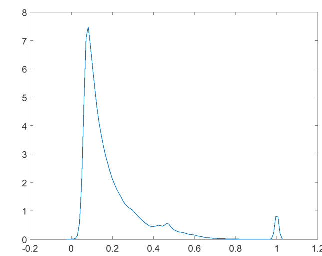
(C) Distribution of γ_{3jt}



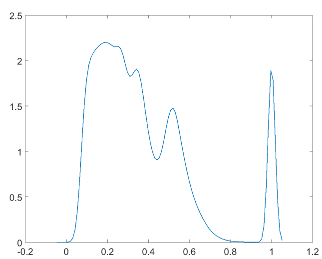
(D) Distribution of γ_{4jt}



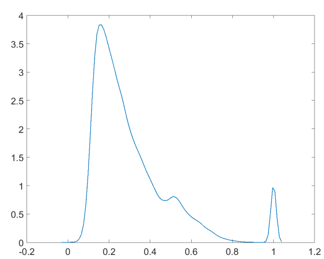
(E) Distribution of γ_{5jt}



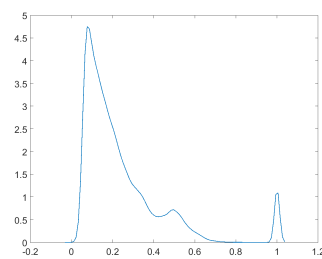
(F) Distribution of γ_{6jt}



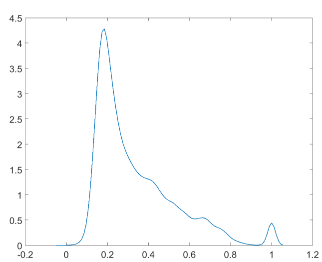
(G) Distribution of γ_{7jt}



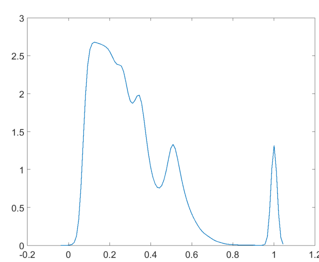
(H) Distribution of γ_{8jt}



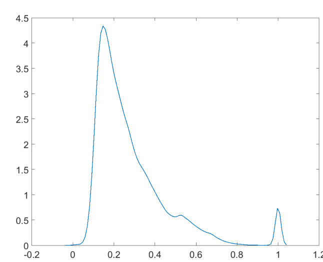
(I) Distribution of γ_{9jt}



(J) Distribution of γ_{10jt}

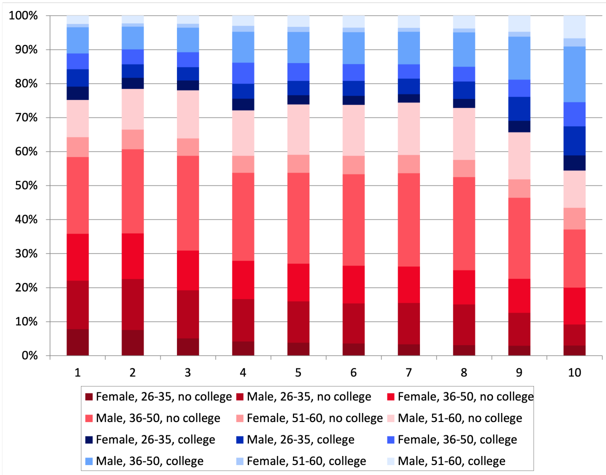


(K) Distribution of γ_{11jt}

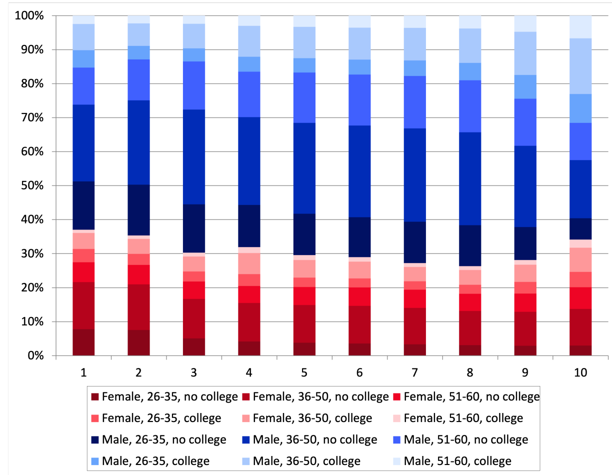


(L) Distribution of γ_{12jt}

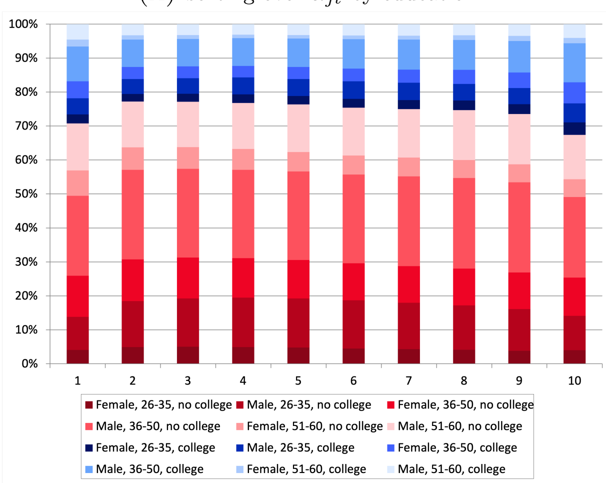
FIGURE F3. The 12 panels show the distribution of the normalized productivity parameter γ_{kjt} for each of the 12 k-groups (equation (5.9)).



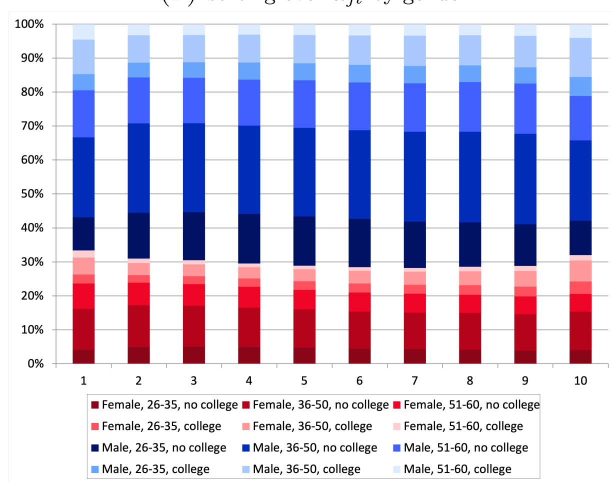
(A) sorting over α_{jt} by education



(B) sorting over α_{jt} by gender



(C) sorting over $\tilde{\theta}_{jt}^{\alpha_{jt}}$ by education



(D) sorting over $\tilde{\theta}_{jt}^{\alpha_{jt}}$ by gender

Figure F4. Sorting of worker types across deciles of the distribution of two separate components of the establishment wage premium: returns to scale α_{jt} and total factor productivity $\tilde{\theta}_{jt}^{\alpha_{jt}}$. This figure shows the employment share of each k-group for each deciles of the establishment-level distribution of α_{jt} and $\tilde{\theta}_{jt}^{\alpha_{jt}}$. In Panels (a) and (c), the k-groups are ordered by education: non-college graduates in red (older workers in lighter red) and college graduates in blue (older workers in lighter blue). In Panels (a) and (c), the k-groups are ordered by gender: women in red (college educated in lighter red) and men in blue (college educated in lighter blue).

	n. unique estab.	n. estab. per firm		n. of workers per estab.		n. of k-groups per estab.		estab. revenue (1,000 UDS)		average wage (USD)		local market share $s_{kj\|gt}$	
commuting zone		mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.
1. North and East Zealand (Copenhagen)	92,731	1.232	3.814	8.454	40.626	2.685	2.356	6,171	61,361	65,148	36,819	0.018	0.071
2. West and South Zealand (Slagelse)	10,718	1.241	4.367	5.816	33.274	2.348	1.897	3,941	55,333	55,326	15,962	0.119	0.214
3. West and South Zealand (Køge)	11,958	1.205	3.241	5.715	19.334	2.383	1.929	3,533	22,430	55,888	17,122	0.112	0.210
4. West and South Zealand (Nykøbing Falster)	4,431	1.249	4.230	5.293	14.390	2.316	1.794	2,729	10,837	51,731	13,878	0.205	0.288
7. Fyn (Odense)	18,879	1.254	3.459	7.285	24.103	2.686	2.251	4,829	29,983	56,571	26,792	0.073	0.160
8. Fyn (Svendborg)	2,934	1.175	3.061	4.953	9.919	2.400	1.917	2,820	8,953	54,654	17,221	0.275	0.317
9. South Jutland (Sønderborg)	5,720	1.243	3.210	8.191	48.529	2.614	2.162	5,614	31,401	54,918	16,683	0.172	0.262
10. South Jutland (Ribe)	2,053	1.183	2.188	5.554	17.850	2.298	1.879	4,179	22,627	52,262	13,968	0.298	0.334
11. South Jutland (Kolding)	9,611	1.254	2.677	7.322	19.109	2.727	2.280	4,924	17,371	56,778	17,735	0.123	0.219
12. Mid-South Jutland (Vejle)	14,565	1.194	2.730	7.820	45.272	2.680	2.258	6,017	58,046	57,835	21,745	0.088	0.181
13. South-West Jutland (Esbjerg)	10,561	1.226	3.130	6.981	22.509	2.590	2.167	5,420	58,499	55,862	16,837	0.111	0.208
14. West Jutland (Herning)	9,517	1.200	3.119	7.040	22.462	2.605	2.156	4,583	22,913	55,664	15,332	0.115	0.208
15. North-West Jutland (Thisted)	2,138	1.207	2.728	6.329	21.196	2.416	1.975	4,009	15,606	54,166	13,972	0.292	0.326
16. East Jutland (Aarhus)	31,814	1.245	3.775	7.399	24.617	2.678	2.271	5,160	53,258	59,102	22,934	0.047	0.123
17. Mid-North Jutland (Viborg)	7,980	1.188	3.828	6.901	47.707	2.493	2.077	4,071	26,117	54,906	15,959	0.137	0.233
19. North Jutland (Aalborg)	23,580	1.208	2.551	6.523	21.000	2.520	2.115	4,499	49,905	55,542	18,251	0.062	0.147
industry													
A. Agriculture, forestry, and fishery	13,499	1.042	0.746	2.302	4.045	1.643	1.246	1,720	2,909	48,810	13,767	0.110	0.206
B. Mining and quarrying	431	1.756	3.568	13.872	62.902	2.767	2.500	35,220	298,848	72,556	101,517	0.315	0.332
C. Manufacturing	20,892	1.174	1.363	18.924	73.662	3.872	2.978	12,355	73,817	60,794	18,468	0.177	0.282
D. Electricity, gas, steam etc.	921	1.263	1.592	15.340	46.976	3.372	2.926	34,650	321,543	73,488	30,898	0.222	0.288
E. Water supply, sewerage etc.	1,954	2.123	3.517	10.479	21.034	3.112	2.306	4,353	14,119	59,114	13,886	0.214	0.296
F. Construction	31,942	1.050	0.708	5.145	14.408	2.298	1.696	2,649	12,075	57,610	14,378	0.032	0.103
G. Wholesale and retail trade	69,175	1.383	5.732	5.514	15.559	2.518	1.992	6,680	36,576	56,683	21,619	0.039	0.106
H. Transportation	15,580	1.270	5.066	11.277	50.020	2.794	2.331	7,665	114,425	57,890	25,777	0.059	0.156
I. Accommodation and food services	15,791	1.236	2.937	3.370	9.242	2.038	1.638	1,488	4,217	48,049	13,443	0.089	0.182
J. Information and communication	15,523	1.174	2.949	10.968	49.839	2.912	2.604	5,163	29,492	76,131	40,250	0.067	0.183
L. Real estate	13,051	1.343	2.305	3.541	8.919	2.080	1.728	1,139	4,436	59,727	25,909	0.045	0.111
M. Knowledge-based services	27,529	1.139	1.298	7.590	30.831	2.753	2.433	2,798	18,008	72,659	47,190	0.070	0.159
N. Travel agent, cleaning etc.	13,777	1.279	2.242	6.724	19.534	2.668	2.325	3,153	12,084	59,338	36,342	0.138	0.240
R. Arts, entertainment, recreation	5,790	1.423	3.026	5.765	14.060	2.799	2.420	1,048	22,416	54,942	19,372	0.099	0.193
S. Other services	13,335	1.126	1.543	4.523	13.985	2.222	1.972	419	2,547	55,563	16,467	0.068	0.143
All local labor markets	259,190	1.227	3.494	7.414	33.071	2.611	2.223	5,199	49,993	59,311	27,048	0.074	0.175

TABLE F1. Establishment characteristics, full sample, all years. Source: Administrative registers, Statistics Denmark. Full population of private sector establishments in Denmark (step 1 in Table E3). Commuting zones computed for 2005 by Eckert, Hejlesen and Walsh (2022), largest city in parentheses. We drop six small islands and we merge Aalborg and Frederikshavn. 1-digit industry classification based on NACE rev. 2. We exclude the public sector, including the health and education sectors. Revenue and average wage at the firm in 2022 USD. Local labor market shares calculated as average share of each k-group workers employed at the establishment over total number of k-group workers in the commuting zone×3-digit industry×year market g.

	n. unique estab.	n. estab. per firm		n. of workers per estab.		n. of k-groups per estab.		estab. revenue (1,000 UDS)		average wage (USD)		local market share $s_{kj\|gt}$	
commuting zone		mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.
1. North and East Zealand (Copenhagen)	20,352	1.203	2.905	13.603	54.919	3.672	2.581	11,148	71,714	68,274	24,494	0.021	0.078
2. West and South Zealand (Slagelse)	2,594	1.198	2.172	9.008	46.099	3.106	2.087	6,915	77,429	57,408	13,925	0.123	0.217
3. West and South Zealand (Køge)	2,823	1.256	3.961	9.000	26.336	3.212	2.113	6,138	31,361	58,367	15,116	0.119	0.217
4. West and South Zealand (Nykøbing Falster)	1,095	1.234	3.769	7.981	18.888	3.028	1.954	4,528	14,376	53,727	12,963	0.212	0.288
7. Fyn (Odense)	4,923	1.254	3.575	11.125	31.529	3.575	2.438	7,969	39,127	58,870	16,999	0.080	0.169
8. Fyn (Svendborg)	745	1.280	6.141	7.356	12.324	3.189	2.086	4,691	11,938	56,857	14,928	0.281	0.320
9. South Jutland (Sønderborg)	1,566	1.249	4.162	12.882	65.503	3.433	2.352	9,352	41,824	57,073	14,425	0.172	0.257
10. South Jutland (Ribe)	510	1.141	1.471	9.010	24.495	3.129	2.118	7,210	31,333	54,685	12,547	0.327	0.336
11. South Jutland (Kolding)	2,637	1.225	1.760	11.245	24.572	3.613	2.485	8,000	22,656	59,639	16,070	0.124	0.217
12. Mid-South Jutland (Vejle)	3,958	1.222	3.855	12.182	60.543	3.587	2.456	10,021	78,260	60,381	18,023	0.092	0.184
13. South-West Jutland (Esbjerg)	2,899	1.227	2.530	10.648	28.233	3.445	2.362	8,952	79,034	58,512	15,138	0.116	0.212
14. West Jutland (Herning)	2,654	1.198	2.821	10.817	29.165	3.464	2.344	7,453	30,365	57,933	13,439	0.119	0.210
15. North-West Jutland (Thisted)	587	1.218	3.812	9.958	28.068	3.217	2.174	6,650	20,664	56,651	12,735	0.299	0.330
16. East Jutland (Aarhus)	8,199	1.241	3.929	11.303	31.179	3.588	2.456	8,625	72,640	61,478	16,908	0.051	0.127
17. Mid-North Jutland (Viborg)	2,077	1.143	2.313	10.737	65.201	3.349	2.254	6,614	28,985	57,435	15,628	0.147	0.241
19. North Jutland (Aalborg)	5,907	1.224	2.994	10.202	27.552	3.412	2.330	7,560	67,593	57,936	16,245	0.066	0.154
industry													
A. Agriculture, forestry, and fishery	2,235	1.047	0.600	3.782	5.723	2.372	1.583	2,840	4,262	50,896	11,757	0.122	0.213
B. Mining and quarrying	140	1.555	2.314	17.892	68.180	3.553	2.662	48,421	357,241	70,750	31,235	0.327	0.333
C. Manufacturing	8,902	1.166	1.151	24.457	84.925	4.632	2.998	16,004	84,927	61,765	14,128	0.178	0.277
D. Electricity, gas, steam etc.	306	1.180	1.064	20.179	57.205	4.095	3.001	64,549	456,296	74,386	33,991	0.218	0.269
E. Water supply, sewerage etc.	431	1.553	1.876	12.983	25.803	3.719	2.517	7,940	19,440	60,679	12,300	0.248	0.301
F. Construction	8,742	1.060	0.812	7.549	17.836	3.005	1.814	3,949	15,334	59,681	12,588	0.035	0.102
G. Wholesale and retail trade	21,217	1.357	4.895	7.916	19.278	3.254	2.156	9,794	45,456	59,433	19,340	0.042	0.108
H. Transportation	4,290	1.320	5.428	16.608	61.499	3.644	2.484	11,220	105,958	59,758	18,708	0.066	0.161
I. Accommodation and food services	2,016	1.210	1.993	5.750	14.367	3.018	2.008	2,709	6,364	51,705	12,648	0.092	0.182
J. Information and communication	3,521	1.212	2.792	18.323	63.986	4.175	2.851	8,926	38,975	76,905	24,703	0.074	0.191
L. Real estate	1,610	1.175	1.128	5.155	12.528	2.922	2.013	2,714	7,538	68,106	28,961	0.055	0.115
M. Knowledge-based services	6,116	1.160	1.109	12.136	40.652	3.912	2.622	4,899	24,755	72,717	24,334	0.072	0.154
N. Travel agent, cleaning etc.	2,697	1.185	1.206	7.918	23.035	3.305	2.284	5,492	17,011	62,117	20,404	0.146	0.244
R. Arts, entertainment, recreation	389	1.085	0.698	9.414	20.773	3.896	2.719	8,397	70,779	60,055	17,245	0.159	0.265
S. Other services	914	1.166	3.365	7.322	16.345	2.951	2.251	2,260	5,253	57,559	16,769	0.109	0.197
All local labor markets	63,526	1.219	3.240	11.591	44.040	3.515	2.427	8,909	62,960	61,787	19,573	0.080	0.180

TABLE F2. Establishment characteristics, by commuting zone, estimation sample, all years. Source: Administrative registers, Statistics Denmark. Restricted sample of establishments with no missing values for the key estimation variables (step 5 in Table E3). Commuting zones computed for 2005 by Eckert, Hejlesen and Walsh (2022), largest city in parentheses. We drop six small islands and we merge Aalborg and Frederikshavn. 1-digit industry classification based on NACE rev. 2. We exclude the public sector, including the health and education sectors. Revenue and average wage at the firm in 2022 USD. Local labor market shares calculated as average share of each k-group workers employed at the establishment over total number of k-group workers in the commuting zone×3-digit industry×year market g.

		IV			OLS		
		β_k	σ_{kg}		β_k	σ_{kg}	
	k-group (k)		CZ 1 (CPH)	Avg. across CZ		CZ 1 (CPH)	Avg. across CZ
1	Female, 26-35, no college	2.977	1.854	1.911	-0.024	2.314	2.117
		[2.216; 3.487]	[1.635; 2.147]		[-0.047; -0.000]	[2.117; 2.413]	
2	Female, 26-35, college	3.806	1.958	2.216	-0.094	2.245	1.831
		[2.317; 4.507]	[1.602; 2.254]		[-0.128; -0.058]	[2.005; 2.355]	
3	Male, 26-35, no college	1.985	2.345	2.292	0.331	2.673	2.285
		[1.871; 2.210]	[2.049; 2.534]		[0.317; 0.339]	[2.440; 2.697]	
4	Male, 26-35, college	4.111	1.697	1.549	0.323	2.004	1.729
		[3.015; 4.620]	[1.527; 1.895]		[0.301; 0.347]	[1.871; 2.053]	
5	Female, 36-50, no college	2.184	1.957	1.929	0.215	2.160	1.920
		[1.903; 2.487]	[1.796; 2.075]		[0.203; 0.225]	[1.990; 2.173]	
6	Female, 36-50, college	3.214	1.704	2.137	0.000	1.842	1.693
		[2.003; 3.573]	[1.526; 1.828]		[0.046; 0.082]	[1.702; 1.887]	
7	Male, 36-50, no college	1.565	2.215	2.025	0.271	2.323	2.108
		[1.514; 1.744]	[1.982; 2.251]		[0.262; 0.275]	[2.134; 2.299]	
8	Male, 36-50, college	2.090	1.748	1.714	0.106	1.840	1.593
		[1.691; 2.333]	[1.611; 1.818]		[0.094; 0.118]	[1.707; 1.841]	
9	Female, 51-60, no college	1.415	1.898	1.926	0.234	2.125	1.938
		[0.751; 1.953]	[1.723; 2.093]		[0.221; 0.244]	[1.980; 2.223]	
10	Female, 51-60, college	0.331	1.544	1.829	0.194	1.675	1.594
		[-0.942; 1.559]	[1.414; 1.710]		[0.168; 0.223]	[1.555; 1.781]	
11	Male, 51-60, no college	1.299	2.103	2.148	0.260	2.245	2.165
		[1.119; 1.458]	[1.879; 2.191]		[0.249; 0.266]	[2.054; 2.289]	
12	Male, 51-60, college	1.316	1.640	1.665	0.173	1.763	1.633
		[0.588; 1.746]	[1.528; 1.768]		[0.154; 0.187]	[1.654; 1.827]	

TABLE F3. Parameter estimates for equation (5.4), OLS and IV. We estimate the parameters separately by k-group. The first column are the point estimates for β_k. The second column shows estimates for the σ_{kg} for the Copenhagen metro area). The third column shows the average σ_{kg} estimate across commuting zones. Bootstrapped 95 percent confidence intervals in square brackets (Hall, 1992).

Source: Administrative registers, Statistics Denmark.

	$log(u_{kj})$	
Commuting zone (reference: North and East Zealand (Copenhagen))		
West and South Zealand (Slagelse)	-1.042	(0.003)
West and South Zealand (Køge)	-1.150	(0.003)
West and South Zealand (Nykøbing Falster)	-1.552	(0.004)
Fyn (Odense)	-0.817	(0.002)
Fyn (Svendborg)	-1.693	(0.005)
South Jutland (Sønderborg)	-1.215	(0.004)
South Jutland (Ribe)	-2.028	(0.007)
South Jutland (Kolding)	-1.005	(0.003)
Mid-South Jutland (Vejle)	-0.942	(0.002)
South-West Jutland (Esbjerg)	-1.083	(0.003)
West Jutland (Herning)	-1.097	(0.003)
North-West Jutland (Thisted)	-1.686	(0.007)
East Jutland (Aarhus)	-0.471	(0.002)
Mid-North Jutland (Viborg)	-1.159	(0.003)
North Jutland (Aalborg)	-0.501	(0.002)
Industry (reference: A. Agriculture, forestry, and fishery)		
B. Mining and quarrying	-0.664	(0.014)
C. Manufacturing	-0.191	(0.005)
D. Electricity, gas, steam etc.	-0.215	(0.008)
E. Water supply, sewerage etc.	-0.604	(0.008)
F. Construction	0.411	(0.005)
G. Wholesale and retail trade	0.407	(0.004)
H. Transportation	0.326	(0.005)
I. Accommodation and food services	-0.169	(0.005)
J. Information and communication	0.394	(0.005)
L. Real estate	0.090	(0.006)
M. Knowledge-based services	0.235	(0.005)
N. Travel agent, cleaning etc.	-0.404	(0.005)
R. Arts, entertainment, recreation	-0.246	(0.007)
S. Other services	-0.302	(0.007)
Log of establishment size (number of workers)	1.775	(0.004)
Log of establishment wagebill (thousands 2022 USD)	-1.526	(0.004)
Log of establishment revenue (thousands 2022 USD)	-0.019	(0.001)
Log of firm size (number of workers)	0.016	(0.000)
Observations	2,360,853	
R^2	0.803	

TABLE F4. OLS of estimated deterministic preferences for amenities $log(u_{kj})$ on k-group, commuting zone, industry, and year indicators, and establishment characteristics (logarithm of firm and establishment size in number of workers, and logarithm of establishment wage bill and revenue). We report coefficients for commuting zone, industry, and establishment characteristics. Robust standard errors in parentheses, $p < 0.01$.

	k-group	IV $\rho_k - 1$	IV $\delta(\rho_k - 1)$	IV δ	IV ρ_k	OLS ρ_k
1	Female, 26-35, no college	0.010	0.010	0.803	1.009	0.990
		[-0.000; 0.017]	[0.002; 0.016]	[0.800; 0.805]	[1.000; 1.017]	[0.986; 0.992]
2	Female, 26-35, college	0.030	0.030		1.031	0.989
		[0.020; 0.040]	[0.020; 0.039]		[1.020; 1.040]	[0.986; 0.993]
3	Male, 26-35, no college	0.010	0.010		1.012	0.992
		[0.004; 0.019]	[0.004; 0.016]		[1.004; 1.019]	[0.991; 0.995]
4	Male, 26-35, college	0.032	0.032		1.030	0.985
		[0.019; 0.038]	[0.021; 0.039]		[1.019; 1.038]	[0.982; 0.988]
5	Female, 36-50, no college	0.020	0.020		1.021	0.982
		[0.011; 0.031]	[0.011; 0.029]		[1.011; 1.031]	[0.980; 0.985]
6	Female, 36-50, college	-0.004	-0.004		1.003	0.995
		[-0.011; 0.018]	[-0.017; 0.011]		[0.989; 1.018]	[0.991; 1.000]
7	Male, 36-50, no college	-0.015	-0.015		0.983	0.983
		[-0.026; -0.006]	[-0.024; -0.005]		[0.974; 0.994]	[0.981; 0.985]
8	Male, 36-50, college	-0.066	-0.066		0.936	1.003
		[-0.082; -0.045]	[-0.083; -0.048]		[0.918; 0.955]	[0.999; 1.008]
9	Female, 51-60, no college	0.011	0.011		1.014	1.003
		[0.002; 0.026]	[0.000; 0.023]		[1.002; 1.026]	[0.997; 1.003]
10	Female, 51-60, college	-0.003	-0.003		1.008	1.000
		[-0.019; 0.040]	[-0.029; 0.028]		[0.981; 1.040]	[1.032; 1.048]
11	Male, 51-60, no college	-0.003	-0.001		0.998	1.041
		[-0.015; 0.009]	[-0.013; 0.009]		[0.985; 1.009]	[0.989; 0.995]
12	Male, 51-60, college	-0.041	-0.041		0.964	0.992
		[-0.054; -0.008]	[-0.058; -0.015]		[0.946; 0.992]	[1.024; 1.040]

TABLE F5. Parameter estimates for the production function, IV. The first two columns are the point estimates for $(\rho_k - 1)$ and $\delta(\rho_k - 1)$ from equation (5.8). The third and fourth columns show the implied values for δ and ρ_k. The fifth column shows the OLS estimate for ρ_k. Bootstrapped 95 percent confidence intervals in square brackets.

	k-group	η_{kjt} Mean	Median	P10	P90
1	Female, 26-35, no college	-5.667	-9.276	-77.774	-1.188
2	Female, 26-35, college	2.131	-6.995	-80.058	100.415
3	Male, 26-35, no college	-13.378	-5.582	-25.648	-1.709
4	Male, 26-35, college	-41.663	-8.602	-74.412	78.132
5	Female, 36-50, no college	-49.373	-7.233	-42.374	-1.283
6	Female, 36-50, college	-24.723	-12.799	-58.967	-2.673
7	Male, 36-50, no college	-4.126	-3.051	-7.844	-1.425
8	Male, 36-50, college	-3.925	-4.519	-9.071	-1.969
9	Female, 51-60, no college	-5.878	-9.465	-59.891	-1.654
10	Female, 51-60, college	-43.820	-8.597	-62.749	-1.660
11	Male, 51-60, no college	-7.524	-4.753	-14.639	-1.884
12	Male, 51-60, college	-7.184	-6.976	-15.970	-2.287

TABLE F6. Moments of the firm-level labor demand elasticities $\eta_{kjt} \equiv F_k^j/\ell_{kj}F_{kk}^j$.

k-group		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Female, 26-35, no college	**1**	0	-46	-100	-51	-138	107	253	64	-132	-90	2,582	23
Female, 26-35, college	**2**	101	0	-68	-33	-12	-229	164	34	3	24	2,989	33
Male, 26-35, no college	**3**	-72	-41	0	1	-70	-250	-131	-8	-85	-56	730	20
Male, 26-35, college	**4**	-79	-33	-218	0	-57	-108	-210	-1	-68	10	394	24
Female, 36-50, no college	**5**	-21	-36	-134	-24	0	46	-46	15	-52	25	1,160	18
Female, 36-50, college	**6**	-804	-89	54	-99	-605	0	503	77	-553	-698	2,989	63
Male, 36-50, no college	**7**	90	-23	-265	-82	11	-326	0	23	-13	192	476	29
Male, 36-50, college	**8**	166	-27	-349	-62	6	-102	-19	0	26	179	1,177	27
Female, 51-60, no college	**9**	-42	-38	-90	-39	-79	-208	88	22	0	-7	1,276	26
Female, 51-60, college	**10**	-164	-58	-107	-53	-315	329	669	109	-231	0	5,815	60
Male, 51-60, no college	**11**	706	29	-275	-12	331	-592	18	6	280	873	0	37
Male, 51-60, college	**12**	-111	-31	-106	-34	-57	-392	50	16	-78	-66	-42	0

TABLE F7. Each cell is the mean Morishima elasticity of substitution calculated across all firms which employ both types of labor.

k-group	$\mathbb{E}[\log w]$	Var($\log w$)	Emp	GCI	EV	EV-V	EV-G
Female, 26–35, No College	3.4657	0.0484	0.307	0.0460	1.000	–	–
Female, 26–35, College	3.7235	0.0650	0.482	0.0168	1.000	–	–
Male, 26–35, No College	3.6604	0.0525	0.630	0.0036	1.000	–	–
Male, 26–35, College	3.8588	0.0709	0.709	0.0034	1.000	–	–
Female, 36–50, No College	3.6436	0.0537	0.497	0.0085	1.000	–	–
Female, 36–50, College	3.9766	0.0858	0.673	0.0041	1.000	–	–
Male, 36–50, No College	3.8212	0.0626	0.737	0.0015	1.000	–	–
Male, 36–50, College	4.1971	0.1061	0.861	0.0011	1.000	–	–
Female, 51–60, No College	3.6352	0.0603	0.422	0.0166	1.000	–	–
Female, 51–60, College	3.9322	0.1687	0.469	0.0164	1.000	–	–
Male, 51–60, No College	3.8014	0.0712	0.684	0.0023	1.000	–	–
Male, 51–60, College	4.2306	0.1527	0.771	0.0019	1.000	–	–

(A) Baseline

$\mathbb{E}[\log w]$	Var($\log w$)	Emp	GCI	EV	EV-V	EV-G
3.7547	0.1247	0.831	0.0013	5.004	1.039	2.965
4.0707	0.1091	0.959	0.0014	5.373	2.673	1.700
3.7672	0.1213	0.578	0.0042	0.801	-0.124	-0.074
4.2131	0.0993	0.996	0.0010	5.064	3.315	0.750
3.7372	0.1248	0.712	0.0014	1.975	-0.192	1.167
4.1800	0.1330	0.956	0.0007	3.280	1.246	1.034
3.8376	0.1495	0.436	0.0104	0.311	0.041	-0.730
4.2655	0.1865	0.732	0.0014	0.529	-0.391	-0.080
3.6985	0.1336	0.344	0.0267	0.743	0.034	-0.291
3.8928	0.2521	0.018	0.7774	0.033	2.347	-3.313
3.8838	0.1994	0.305	0.0354	0.230	0.338	-1.107
4.4105	0.2737	0.300	0.0426	0.149	0.196	-1.047

(B) CF A: Equal Deterministic Amenities

k-group	$\mathbb{E}[\log w]$	Var($\log w$)	Emp	GCI	EV	EV-V	EV-G
Female, 26–35, No College	3.3622	0.1213	0.056	0.5104	0.285	-0.254	-0.461
Female, 26–35, College	2.7037	2.9818	0.027	0.7299	0.122	-0.465	-0.413
Male, 26–35, No College	3.5762	0.0517	0.665	0.0025	1.170	-0.023	0.193
Male, 26–35, College	2.3218	4.3512	0.022	0.7788	0.065	-0.482	-0.452
Female, 36–50, No College	3.5830	0.0563	0.374	0.0235	0.571	-0.074	-0.355
Female, 36–50, College	3.6219	0.6733	0.094	0.3474	0.093	-0.379	-0.529
Male, 36–50, No College	3.8275	0.0624	0.899	0.0005	7.023	4.041	1.981
Male, 36–50, College	4.1752	0.1018	0.809	0.0018	0.668	-0.149	-0.183
Female, 51–60, No College	3.7038	0.0541	0.792	0.0014	15.385	5.129	9.256
Female, 51–60, College	4.9152	0.0814	0.997	0.0020	***	***	***
Male, 51–60, No College	3.8780	0.0750	0.938	0.0005	40.044	26.647	12.397
Male, 51–60, College	4.4303	0.1281	0.958	0.0011	39.151	33.990	4.161

(C) CF B: Equal Preferences

$\mathbb{E}[\log w]$	Var($\log w$)	Emp	GCI	EV	EV-V	EV-G
4.0762	0.1148	0.693	0.0034	2.663	0.185	1.478
4.4695	0.1758	0.915	0.0031	3.367	1.498	0.869
3.9758	0.0735	0.759	0.0017	1.341	-0.086	0.427
4.4400	0.1454	0.953	0.0019	2.105	0.885	0.220
4.0650	0.0914	0.694	0.0025	1.615	-0.112	0.727
4.2962	0.2066	0.863	0.0017	1.595	0.258	0.337
3.9379	0.0696	0.772	0.0013	0.963	-0.108	0.071
4.1770	0.1032	0.866	0.0013	0.824	-0.122	-0.054
3.9331	0.0913	0.521	0.0084	1.246	-0.289	0.536
3.2847	0.1388	0.417	0.0236	0.607	0.478	-0.871
3.8842	0.0665	0.709	0.0021	0.936	-0.132	0.068
3.9909	0.1157	0.734	0.0026	0.683	-0.146	-0.171

(D) CF C: Equal Match Productivities

k-group	$\mathbb{E}[\log w]$	Var($\log w$)	Emp	GCI	EV	EV-V	EV-G
Female, 26–35, No College	3.6602	0.0942	0.333	0.0433	0.930	-0.088	0.019
Female, 26–35, College	3.5511	0.2040	0.278	0.0737	0.500	-0.220	-0.280
Male, 26–35, No College	3.6964	0.1169	0.665	0.0049	0.659	-0.208	-0.132
Male, 26–35, College	3.6246	0.1306	0.492	0.0133	0.428	-0.350	-0.223
Female, 36–50, No College	3.6927	0.1461	0.422	0.0150	0.654	-0.136	-0.210
Female, 36–50, College	3.6528	0.1934	0.393	0.0221	0.366	-0.301	-0.333
Male, 36–50, No College	3.7047	0.1554	0.899	0.0021	0.528	-0.316	-0.156
Male, 36–50, College	3.6974	0.1668	0.667	0.0022	0.323	-0.491	-0.186
Female, 51–60, No College	3.5801	0.2518	0.333	0.0325	0.631	0.012	-0.382
Female, 51–60, College	3.0738	0.4954	0.395	0.0251	0.482	0.394	-0.912
Male, 51–60, No College	3.6368	0.2100	0.590	0.0036	0.519	-0.224	-0.257
Male, 51–60, College	3.6573	0.2481	0.590	0.0044	0.340	-0.285	-0.374

(E) CF D: Equal Firm Productivity

$\mathbb{E}[\log w]$	Var($\log w$)	Emp	GCI	EV	EV-V	EV-G
3.4832	0.0492	0.317	0.0419	1.019	-0.012	0.031
3.7362	0.0640	0.495	0.0153	1.020	-0.006	0.026
3.6830	0.0540	0.640	0.0033	1.012	-0.022	0.034
3.8697	0.0709	0.718	0.0032	1.003	-0.011	0.015
3.6625	0.0536	0.507	0.0079	1.015	-0.019	0.034
3.9902	0.0853	0.682	0.0038	1.006	-0.011	0.017
3.8436	0.0624	0.744	0.0014	1.004	-0.020	0.024
4.2153	0.1065	0.865	0.0011	0.994	-0.015	0.009
3.6723	0.0615	0.434	0.0151	1.029	-0.037	0.066
4.0204	0.1648	0.476	0.0155	1.040	-0.124	0.165
3.8345	0.0711	0.693	0.0022	1.017	-0.025	0.042
4.2622	0.1518	0.779	0.0019	1.012	-0.021	0.033

(F) Monopsony

TABLE F8. Wage moments, employment, concentration, and welfare by k-group across baseline and counterfactual economies. Columns: $\mathbb{E}[\log w]$ is the employment-weighted mean log wage; Var($\log w$) is the employment-weighted variance of log wages; Emp is the employment rate; GCI is the generalized concentration index; EV is equivalent variation relative to the baseline; EV-V is the component due to changes in the value of matching; EV-G is the component due to changes in concentration. By construction, EV-V + EV-G = EV − 1. Counterfactuals: CF A sets job-amenity utilities to their mean; CF B sets preference parameters (β_k, σ_{kg}) to their means (group 10 omitted as unreliable); CF C sets worker–firm match productivity $(\tilde{\gamma}_{kj}, \rho_k)$ to their means; CF D sets firm productivity and returns-to-scale $(\tilde{\theta}_j, \alpha_j)$ to their medians; Monopsony fixes each firm's labor supply elasticity to $\beta_k \sigma_{kg}$. We exclude one entry for k-group 10 because the estimated wage-preference parameter β for this group is not statistically significant and close to zero, leading to unreliable results.

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