Earnings Inequality in Production Networks*

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Abstract

We explore the importance of production networks for firm earnings premia and labor shares. Using Chilean employer-employee and firm-to-firm trade data, we show that firms with better access to buyers and suppliers tend to have higher earnings premia and lower labor shares. Workers also benefit more when firm growth is driven by customer demand instead of supplier cost. We develop and estimate a model capable of rationalizing these facts, featuring firm labor market power, firm-to-firm trade linkages, and a flexible labor-materials substitution elasticity. Counterfactual simulations show that networks are key for explaining firm heterogeneity in earnings premia and labor shares.

Keywords: Earnings inequality, production networks, monopsony power.

JEL Codes: J31, F16.

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1 Introduction

How do firms matter for worker outcomes? A growing body of research has investigated this question along three dimensions: how firms differ in the wages paid to otherwise identical workers (earnings premia heterogeneity), how firms differ in the share of revenue allocated to labor (labor share heterogeneity), and how changes in firm performance affect worker earnings (firm-to-worker pass-through). These three branches of the literature have found, respectively, that differences in firm earnings premia are important for explaining inequality in worker earnings (e.g. Abowd et al. (1999), Card et al. (2013), Bonhomme et al. (2019), Lamadon et al. (2022)), that heterogeneity in labor shares across firms matters for understanding trends in aggregate labor shares (e.g. Autor et al. (2020), Kehrig and Vincent (2021)), and that workers gain from improvements in firm productivity and demand, albeit incompletely (e.g. Guiso et al. (2005), Kline et al. (2019), Dhyne et al. (2022)).

This has motivated the need for a theoretical framework capable of explaining these empirical findings. Existing frameworks that have been proposed explain some of the findings, but are inconsistent with others. Models of firm labor share heterogeneity abstract from firm-specific earnings premia since they assume perfectly competitive labor markets (e.g. Autor et al. (2020)). On the other hand, models of earnings premia heterogeneity and firm-to-worker pass-through assume value-added or Cobb-Douglas production functions (e.g. Berger et al. (2022), Lamadon et al. (2022), Dhyne et al. (2022)) which impose constant labor cost shares for each firm. The implications for firm-to-worker pass-through are more mixed. With a value-added production function, constant labor supply elasticities, and variable input costs, growth in value-added per worker is passed through one-for-one into growth in worker earnings (complete pass-through). Perhaps less well-known – and rarely tested – is that these frameworks also tend to feature symmetric pass-through: whether firms grow due to demand or cost shocks does not matter for how workers gain from such growth (symmetric pass-through).

In this paper, we empirically test and statistically reject the assumption of symmetric pass-through of demand and cost shocks and propose a novel model that is capable of accounting for this combined set of facts. The model emphasizes three key features. First, firms have labor market power so that identical workers earn different wages at different firms as in Manning (2003), Card et al. (2018), Lamadon et al. (2022), Azar et al. (2022), Chan et al. (2025), and Kroft et al. (2025). Second, firms source materials from and sell to other firms in a production network with heterogeneous firm-to-firm linkages as in Lim (2019), Huneeus (2019), Bernard et al. (2022), and Dhyne et al. (2022). Third, the production function combines heterogeneous labor and heterogeneous materials in a flexible way so that labor shares of cost are variable and heterogeneous across firms.

These combined features generate heterogeneity in earnings premia and labor shares across firms, which stems from several sources. As is traditionally emphasized in the literature, firms differ in terms of their innate characteristics. Van Reenen (1996), Kline et al. (2019), and Lamadon et al. (2022) consider heterogeneity in firm earnings premia as arising in part from differences in firm productivities.[1] Similarly, Autor et al. (2020) and Gouin-Bonenfant (2022) emphasize heterogeneity in firm markups and productivities, respectively, to explain differences in labor shares across firms, while Kehrig and Vincent (2021) emphasize the importance of demand-side heterogeneity. Perhaps more novel, firms may differ because of heterogeneity in their network connections with other firms, leading to variation in the cost of materials and customer demand. While these network linkages have been shown to be crucial for explaining differences in firm outcomes that are correlated with earnings premia and labor shares such as size (e.g. Bernard et al. (2022)), their implications for labor market outcomes directly are less well-understood. Importantly, we show that earnings premia vary across firms, *conditional on firm size*, if there is heterogeneity in the labor share (which is what we find empirically).

Our empirical analysis is based on a novel administrative dataset that links matched employer-employee data to firm-to-firm transactions from Chile. As is well-known, there is

[1] Dunne et al. (2004), Faggio et al. (2010), and Barth et al. (2016) also interpret trends in wage dispersion as being related to productivity dispersion across industries and firms.

high income inequality in Chile. For instance, earnings premia and labor shares of value-added are 2.2 and 3.6 times higher respectively for firms at the 75th percentile of the corresponding distribution compared with firms at the 25th percentile. Using this linked dataset, we document three new stylized facts that motivate our focus on production networks and labor market outcomes. First, using two-way fixed effects models for earnings (as in Bonhomme et al. (2019)) and firm-to-firm transactions (as in Bernard et al. (2022)), we show that firms with greater access to larger customers and more efficient suppliers have higher earnings premia. Second, these firms also have lower labor shares of value-added and cost. Third, using a shift-share design following Hummels et al. (2014) and Garin and Silvério (2022), we find that increases in downstream demand and improvements in upstream production efficiency in a firm's supply chain raise earnings for the firm's workers, but less than one-for-one relative to growth in firm size. In addition, downstream demand shocks increase worker wages more than upstream efficiency shocks conditional on the same growth in firm size. In particular, we estimate that following an increase in customer demand, average wages increase by 89 cents for every dollar increase in sales, whereas the corresponding increase following a reduction in supplier cost is 69 cents. While the findings on *incomplete pass-through* of demand shocks are similar to Dhyne et al. (2022), we believe this is the first paper to demonstrate *asymmetric pass-through* with respect to direct and indirect demand and cost shocks. Taken together, these three facts highlight that production network linkages play an important role in shaping heterogeneity in earnings premia and labor shares. In addition, they provide direct evidence that production network linkages matter for firm-to-worker pass-through, establishing not only that workers fail to fully capture the benefits of firm growth, but that the source of firm growth in the production network matters for how workers gain.

We then show that our model rationalizes these stylized facts through the following mechanisms. First, a firm's marginal revenue product of labor (MRPL) is higher when there is more demand for its output or when its suppliers are more efficient (as long as labor and materials are not too substitutable). With firm labor market power, a higher MRPL translates into higher wages. Hence, greater demand and lower supplier cost translate into higher earnings premia. Second, greater supplier efficiency implies a lower cost of materials. Combined with the preceding mechanism, greater customer demand and supplier efficiency imply a higher relative cost of labor to materials, which translates into lower labor shares of cost and value-added when labor and materials are substitutes. Finally, the preceding mechanisms operate for changes within firms as well: following a positive customer demand or supplier efficiency shock, a firm's earnings premium and its relative cost of labor to materials both rise. The latter change leads to a reduction in spending on labor versus materials when the two inputs are substitutes, which partially offsets the increase in the firm's earnings premium. Furthermore, this substitution effect is stronger for upstream efficiency shocks than for downstream demand shocks, since only the former directly affects the relative cost of labor to materials. Thus, workers benefit more from demand shocks compared with supplier efficiency shocks conditional on the same growth in firm size. In contrast, with Cobb-Douglas or value-added production functions, there is symmetric pass-through.

Having shown that our model can replicate the stylized facts, we then use it to quantify how production network heterogeneity contributes to heterogeneity in firm earnings premia and labor shares. We first show how to identify and structurally estimate the model's parameters using the Chilean administrative data. Two parameters of the model are particularly important: the elasticity of substitution between labor and materials (ϵ) and the labor supply elasticity (γ).

First, it is well-known that the labor-materials substitution elasticity ϵ can be identified from the relationship between a firm's relative expenditures on these inputs and their relative prices. However, the literature offers little guidance for how input prices should be aggregated when both wages and material prices are heterogeneous within firms. We show how to construct model-consistent price aggregates for labor and materials in the presence of such heterogeneity. The labor price aggregate is obtained as the firm effect in a decomposition of worker earnings into worker and firm effects (Bonhomme et al. (2019)). This improves on existing approaches that treat the labor price aggregate as an average firm wage (as in Doraszelski and Jaumandreu (2018)) since this may be confounded by compositional differences in worker quality across firms. Similarly, the materials price aggregate is constructed from seller effects obtained from a decomposition of firm-to-firm transactions into buyer and seller effects (Bernard et al. (2022)). Our approach allows us to infer firm heterogeneity in the material price even though we do not observe the prices directly in our data and improves on approaches that treat material prices as an industry characteristic rather than firm characteristic (as in Oberfield and Raval (2019)). Using our approach and an instrumental variables (IV) strategy which follows Doraszelski and Jaumandreu (2018), we estimate $\epsilon = 1.5$, indicating gross substitutability of labor and materials, which is consistent with findings by Chan (2023). We also statistically reject the hypothesis of Cobb-Douglas production functions. Importantly, if we instead follow the common practice in the literature of using *average prices* instead of our preferred price indices in the IV estimation, we find an elasticity much closer to $\epsilon = 1$. This demonstrates the importance of accounting for heterogeneous inputs in a model-consistent way when estimating the elasticity of substitution.

Second, existing approaches in the literature identify the labor supply elasticity (γ) from the pass-through into worker earnings following changes in firm value-added (as in Guiso et al. (2005) and Lamadon et al. (2022), for example). However, we show that this approach is valid only when a firm's wage bill is proportional to its value-added, a restriction that is generally violated when firms have heterogeneous labor cost shares. In such cases, a firm's labor cost share enters as a residual in the relationship between worker earnings and value-added. Since typical instruments for value-added are also likely to be correlated with labor shares, this violates the exclusion restriction underlying IV approaches to estimating the pass-through from firm value-added into worker earnings. Hence, we extend this identification strategy to allow for labor cost share heterogeneity, which motivates identifying γ from the pass-through into worker earnings following changes in firm wage bills rather than value-added. Using this approach, we estimate $\gamma = 5.5$.

Finally, we use the estimated model to investigate what drives the variances of worker earnings, firm earnings premia, and labor shares. We first show that our estimated model provides a good fit to these outcomes in the data. We then solve for counterfactual equilibria in which heterogeneity in different sets of model primitives – including a firm's network linkages – is removed and use these simulations to quantify the importance of production network heterogeneity for the labor market variances and covariances. We find that production network heterogeneity is a key driver of firm heterogeneity in earnings premia and labor shares, accounting for 13% of the overall variation in worker earnings, one-third of the variation in firm-specific earnings premia, and one-quarter of the variation in labor shares of value-added. We also find that production network heterogeneity explains half of the positive covariance between firm size and earnings premia and two-thirds of the negative covariance between firm size and labor value-added shares. Furthermore, restricting the production function to be Cobb-Douglas ($\epsilon = 1$) greatly overstates the importance of production network heterogeneity for explaining earnings inequality. Taken together, our findings highlight the importance of firm-to-firm production network linkages for earnings premia heterogeneity, labor share heterogeneity, and the pass-through of demand and cost shocks into changes in worker earnings. Our findings also stress the need to move away from value-added or Cobb-Douglas production functions to build a unified framework that can address all three issues simultaneously.

To our knowledge, there are only five other papers that study linked employer-employee and firm-to-firm transactions data. First, Adao et al. (2020) use data from Ecuador to measure the effects of international trade on individual-level factor prices. Second, Demir et al. (2024) study the effects of trade-induced product quality upgrading on wages in Turkey. Both of these analyses assume a market price for skill and focus on the effects of trade shocks. In contrast, we allow for imperfect competition in labor markets and use our data to speak to the role of the production network itself in shaping earnings inequality. Third, Alfaro-Ureña et al. (2020) adopt an event study research design to examine the effects on worker earnings in Costa Rica when a local firm starts interacting with multinationals. In contrast, we use our data to study the determinants of both worker-level earnings and aggregate outcomes such as earnings inequality, which requires a general equilibrium model. Fourth, Dhyne et al. (2022) combine VAT data on the universe of firms in Belgium with linked employer-employee data for a random subsample of workers to estimate how foreign demand shocks affect earnings of stayers at a firm. In contrast, our data contain information on the universe of formal sector workers, which allows us to use these data not only for the analysis of pass-through but also for general equilibrium economy-wide counterfactuals. Finally, Cardoza et al. (2024) provide empirical evidence from the Dominican Republic that workers are more likely to move to a customer or supplier of their original employer than to an unrelated firm and that movers between buyers and suppliers experience larger earnings increases than other movers. This form of interaction between the production network and the labor market is beyond the scope of our analysis due to the challenge in allowing for directed worker mobility in our theoretical framework, but we view this as an interesting area for future theoretical work.

The remainder of the paper is organized as follows. Section 2 describes the Chilean data that we use in the paper and Section 3 presents the three stylized facts that motivate our focus on production networks. Section 4 develops a structural model of labor markets and production networks, while Section 5 provides theoretical propositions characterizing how the model is able to rationalize the stylized facts. Section 6 discusses identification and estimation of the model's parameters, while Section 7 presents counterfactual simulations using the estimated model. Finally, Section 8 concludes.

2 Data

Our empirical analysis relies mainly on three administrative datasets from the Internal Revenue Service in Chile (IRS, or SII for its acronym in Spanish), covering the entire formal private sector.[2] The first is an employer-employee dataset (IRS tax forms 1887 and 1879), which reports annual earnings from each job that a worker has in a given year, including wages, bonuses, tips, and other sources of labor income deemed taxable by the IRS between 2005 and 2022. We adjust earnings to include social security payments. The second is a firm-to-firm trade dataset (IRS tax forms 3323) based on value-added tax (VAT) records between 2005 and 2010. Each firm in this dataset reports the full list of its buyers and suppliers, as well as the total value of transactions with each buyer and supplier at semi-annual frequency. We aggregate this data to annual frequency and measure transactions inclusive of taxes using the flat VAT rate of 19% that was in effect in Chile during the sample period. The third is a firm balance sheet dataset (IRS tax form 29), which we use to measure total sales and material cost for each firm between 2005 and 2022. Firms are assigned a unique tax ID in each of these three datasets, which facilitates their merging, and we henceforth define a firm as a tax ID.[3]

To prepare the data for use in our analysis, we first clean the employer-employee and firm-to-firm trade data as described in Appendix A. We refer to these cleaned datasets, respectively, as the *baseline employer-employee dataset* and the *baseline firm-to-firm dataset*. We then define three samples that will be used in different parts of the paper.

The first, which we refer to as the *stayers* sample, restricts the baseline employer-employee dataset to workers observed with the same employer for at least 8 consecutive years and to employers that have at least 10 stayers in each year. We will use this sample to estimate the labor supply elasticity, which relies on the pass-through of firm shocks into worker earnings.

[2] We convert all nominal variables to real 2015 dollars.

[3] As all tax forms are reported at the headquarter-level, plant-level information is not available. Furthermore, while it is possible that a firm has several tax IDs, information that allows us to observe firm ownership is not available.

The restrictions on this sample will allow for a flexible specification of how worker earnings evolve over time at a given firm and ensure a sufficient sample size to perform analyses at the firm level. We also omit the first and last years of workers' employment spells to avoid concerns over exit and entry into employment during the year, which might confound our measure of earnings.

The second, which we refer to as the *movers* sample, restricts the baseline employer-employee dataset to workers observed at multiple firms over time. We will use this sample to decompose worker earnings into worker and firm effects. Following previous work and motivated by concerns about limited mobility bias, we restrict the movers sample to firms with at least two movers (Lamadon et al., 2022). In addition, as in the previous literature (Abowd et al., 1999; Lamadon et al., 2022), we restrict this sample to firms that belong to the largest connected set of firms, which account for 99.9% of workers in our data.

Finally, we merge the baseline employer-employee and firm-to-firm datasets, referring to this as the *baseline firm-level dataset*. We implement this merge at the firm-year level and thus exclude firms that do not have information in either the employer-employee or the firm-to-firm dataset. We will use this sample primarily to estimate the elasticity of substitution between labor and materials.

Appendix Table A.I compares the size of the various samples described above, while Appendix Table A.II provides detailed summary statistics. We compare here the main differences between the different samples used in the paper.

Comparing the stayers sample (Column 3) to the baseline sample (Column 1), we note that firms in the stayer sample are largely similar to firms in the baseline employer-employee dataset on a number of dimensions: value-added per worker, materials share of sales, and intermediate sales as a share of total sales. There are some differences, however, in worker and firm characteristics across the samples. In the stayers sample, workers are slightly older and have higher earnings and firms have higher labor shares, employment, and value-added than in the baseline sample. The large difference in the number of workers comes from conditioning on firms with at least least 10 stayers in each year.

Next, we compare firms from the firm baseline sample (Column 5), which combines employer-employee and firm-to-firm characteristics, with firms from the employer-employee baseline sample (Column 1). We see that while the firms across the samples are similar in some dimensions (median age, median labor share, median value-added per worker, median wage bill per worker), the firms in Column (5) are different in several ways. Firms from the firm baseline sample (Column 5) are larger both in terms of number of workers and value-added compared to the firms from the baseline employer-employee sample (Column 1). In the case of the number of workers, this difference is substantial. The average firm from the firm baseline sample has three times the number of workers of the average firm from the employer-employee baseline sample. We also see that workers' average earnings are relatively smaller in the firm baseline sample relative to the employer-employee baseline sample. We note that these differences between samples are natural since the firm baseline sample conditions on having more information (both firm-to-firm and employer-employee), which typically holds for larger firms.

Lastly, we see that in the firm baseline sample, the median number of buyers is 8, the median number of suppliers is 36 and the median number of workers is 7. These numbers are quite similar to the ones from the literature. For example, in the Belgium data, the median number of buyers is 9 and the median number of suppliers is 33 (Dhyne et al., 2021).[4] While the number of suppliers is higher than the number of workers, this is consistent with the stylized fact that the materials share (0.61 in our sample) is around twice the labor share (0.34 in our sample).

[4]Although not documented in their paper, Dhyne et al. (2021) find that the median number of workers in their sample is 3.5.

3 Motivating Evidence

Earnings inequality in Chile is severe. For instance, in the average year of our sample, workers at the 90^{th} percentile of the earnings distribution have earnings that are almost seven times higher than workers at the 10^{th} percentile of the distribution. In comparison, the average 90-10 ratio among all other OECD members is just slightly over four (based on data from the OECD Income Inequality database). Similarly, the earnings Gini coefficient in Chile of around 0.5 is the highest among OECD members and lies above the 90^{th} percentile across all countries during this period based on data from the World Bank.

There is also substantial heterogeneity in the extent to which firms allocate revenue to labor versus other productive inputs. To fix ideas, suppose that firms produce output using labor and materials. Consider the share of labor in total production cost (labor plus material cost) and in value-added (sales minus material cost). We observe substantial differences across firms in both of these labor shares. For example, firms at the 75^{th} percentile of the labor cost share distribution spend eight times more of their production cost on workers compared with firms at the 25^{th} percentile (88% versus 11%). Similarly, the 72-25 ratio for the labor value-added share distribution is around 3.6 (61% versus 17%).

To motivate our focus on the role of production network linkages in explaining this observed heterogeneity in earnings and labor shares, we now present three new stylized facts about the interaction between these labor market outcomes and the production network in Chile. We begin with some definitions of key variables that we use to establish these facts.

3.1 Measuring firm earnings premia and network access

To obtain a measure of a firm's premium on earnings, we first decompose the log earnings of worker m at firm i and time t as:

$$\log w_{imt} = \theta_i x_m + \log f_{it} + \hat{x}_{imt} \tag{3.1}$$

where x_m is a worker fixed effect, f_{it} is a time-varying firm effect, θ_i allows for worker-firm interactions, and $\hat{x}_{imt}$ is an orthogonal residual. As we discuss below, this decomposition is consistent with the structural model that we develop in the paper and can be viewed as an extension of the well-known earnings model in Abowd et al. (1999) to allow for worker-firm interactions (as in Bonhomme et al. (2019)) and time-variation in the firm effect. In particular, the firm effect f_{it} provides a measure of an employer's premium on wages after adjusting for differences in worker composition, which would otherwise be reflected in measures such as the average wage at the firm.

We estimate the decomposition (3.1) using the movers sample of the employer-employee dataset following a procedure that we describe in detail below when discussing the estimation of our structural model (see section 6.2.2). We find substantial variation in firm earnings premia. For instance, firms at the 75^{th} percentile of the distribution have earnings premia that are 2.2 times greater than firms at the 25^{th} percentile of the distribution. We also find that variation in firm earnings premia explains around 11% of the total variation in worker earnings, while the covariance between firm earnings premia and worker effects explains around 20%.[5] This highlights the importance of differences in firm earnings premia for explaining differences in worker earnings more broadly.

Next, to measure a firm's access to customers and suppliers in the production network, we first decompose log sales by a seller j to a buyer i at time t as follows:

$$\log r_{ijt} = \log d_{it} + \log s_{jt} + \log e_{ijt} \tag{3.2}$$

where d_{it} is a buyer effect, s_{jt} is a seller effect, and e_{ijt} is an orthogonal residual.[6] Intuitively, firms with larger buyer effects tend to spend more on inputs from their suppliers conditional on their suppliers' characteristics, while firms with larger seller effects tend to sell more to

[5]This variance decomposition follows Lamadon et al. (2022). Using US data, they find shares of 4.3% (less than half of our value for Chile) and 13.0% (about two-thirds of our value for Chile), respectively. See Online Appendix A for a formal discussion of the decomposition

[6]This decomposition is also studied by Bernard et al. (2022) using firm-to-firm trade data from Belgium.

their customers conditional on their customers' characteristics.

We estimate this decomposition using the baseline firm-to-firm dataset following a procedure that we describe in detail below when discussing the estimation of our structural model (see section 6.2.3). We then construct measures of a firm's *downstream access* (D_{it}^{net}) and *upstream access* (S_{it}^{net}):

$$D_{it}^{net} \equiv \sum_{j \in \Omega_{it}^{C}} d_{jt} e_{jit}, \qquad S_{it}^{net} \equiv \sum_{j \in \Omega_{it}^{S}} s_{jt} e_{ijt} \tag{3.3}$$

where Ω_{it}^{C} and Ω_{it}^{S} denote the set of firm i's customers and suppliers, respectively. Intuitively, D_{it}^{net} summarizes the extent to which firm i is connected to customers that have high demand for intermediate inputs, while S_{it}^{net} summarizes the extent to which the firm is connected to suppliers that tend to have high sales to other firms in the production network. As we show in the structural model that we develop below, D_{it}^{net} and S_{it}^{net} are sufficient statistics for the relevance of the production network for firms' wage-setting decisions.

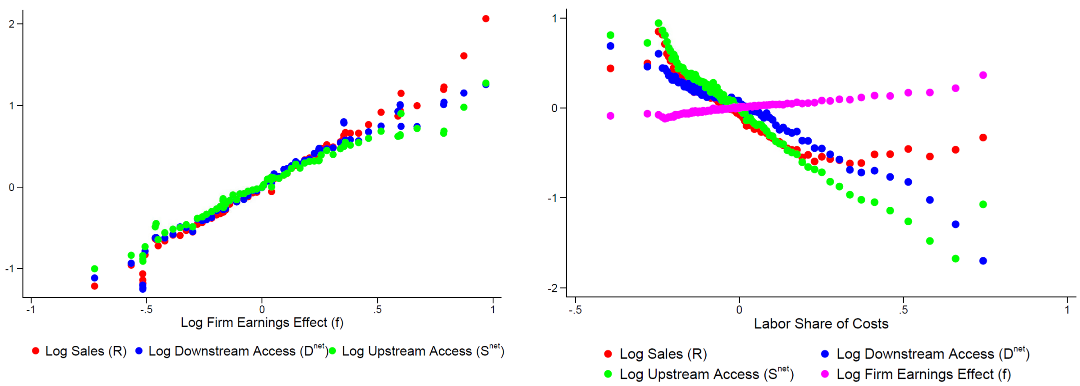
3.2 Stylized facts

With these measures of firm earning premia and network access in hand, we now turn toward our three new stylized facts.

Fact 1: Firms that have greater downstream and upstream access in the production network tend to have higher earnings premia.

The relationship between firm earnings premia and network access is documented in the left panel of Figure 1, which shows bin scatter plots of a firm's sales R_{it}, downstream access D_{it}^{net}, and upstream access S_{it}^{net} against its earnings premium f_{it}, where all variables are residualized by industry-municipality-year fixed effects. Evidently, firms with greater access to customers and suppliers in the production network have higher earnings premia, suggesting a role for production network heterogeneity in explaining earnings inequality. As expected, we also find that larger firms tend to pay higher wages.

Figure 1: Firm earnings premia, labor cost shares, network access, and sales



Notes: All plots are generated using the bin scatter program provided by Michael Stepner: https://michaelstepner.com/software. The firm earnings effect is measured as f_{it} from the worker earnings decomposition in equation (3.1). The network access measures D^{net} and S^{net} are as defined in (3.3). All variables are parsed of industry-municipality-year means. We use the firm baseline sample defined in Section 2.

Fact 2: Firms that have greater downstream and upstream access in the production network tend to have lower labor shares of production costs and lower labor shares of value-added.

The right panel of Figure 1 shows bin scatter plots of a firm's sales R_{it}, downstream access D_{it}^{net}, and upstream access S_{it}^{net} against its labor share of cost $s_{it}^{L/C}$, where again all variables are residualized by industry-municipality-year fixed effects. Here, we see that firms with greater network access tend to have lower labor shares of cost, again suggesting a role for production network heterogeneity in explaining differences in labor shares. We find similar patterns when considering a firm's labor share of value-added instead of its labor cost share. Although there is some non-monotonicity in the middle of the labor share distribution, the overall correlation between labor shares of value-added and both our downstream and upstream access measures is negative. In addition, we find a negative relationship between firm size and both labor share measures, which is consistent with recent evidence documented by Autor et al. (2020).[7]

A natural question that arises is to what extent the importance of production networks for earnings premia and labor shares reflects firm size versus holding conditional on firm size. In Appendix E.1, we present a descriptive analysis that removes the influence of firm size. Our key findings are that there is a negative correlation between the firm earnings premia and firm share of labor costs and upstream access, conditional on firm size. These results are consistent with our theoretical model which show that this is driven by a pure substitution effect: controlling for changes in sales effectively controls for the scale effect associated with better access to suppliers, so all that is left is the substitution effect, which is negative if labor and materials are substitutes (which is what we find empirically). On the other hand, there is no relationship between these labor measures and downstream access, once firm size is controlled for. Intuitively, this is because downstream access only reflects scale effects, which are entirely captured by firm sales. Once again, these results are consistent with the prediction of our model.

Fact 3: Higher demand for a firm's customers and lower input costs for a firm's suppliers raise earnings for the firm's workers. Earnings increase less than one-for-one with firm size, with stronger effects from demand versus cost shocks conditional on the same growth in firm size.

To establish this fact, we require firm-level demand and input cost shocks. For this, we rely on shocks to export demand and imported intermediate costs for Chilean firms. Let market $h \equiv \{p, c\}$ denote a product (p) by country (c) pair, with $p(h)$ and $c(h)$ denoting the product and country corresponding to market h. We first measure the log change in the value of exports (imports) of product $p(h)$ by country $c(h)$ to (from) all destinations excluding Chile between years t' and t, $\hat{X}_{ht't}$ ($\hat{M}_{ht't}$). As highlighted by Garin and Silvério

[7] We also find a weakly positive relationship between a firm's labor share and its earnings effect. In comparison, Kehrig and Vincent (2021) find that average wages are essentially unrelated to a firm's labor share of value-added, although as we show in our structural model below, it is the firm earnings premium rather than the average wage that is relevant for a firm's labor shares of cost and value-added.

(2022), these changes in trade flows may reflect both *idiosyncratic* shocks that are specific to market h as well as *aggregate* shocks that are either common across all countries that trade product $p(h)$ or across all products traded by country $c(h)$. To separate these two sets of shocks, we project the changes in log trade flows on product and country fixed effects:

$$\hat{X}_{ht't} = \delta^{X}_{p(h)} + \mu^{X}_{c(h)} + \tilde{\hat{X}}_{ht't}, \qquad \hat{M}_{ht't} = \delta^{M}_{p(h)} + \mu^{M}_{c(h)} + \tilde{\hat{M}}_{ht't} \tag{3.4}$$

We then construct *direct* export demand and import cost shocks for Chilean firm i by aggregating the idiosyncratic components of the trade flow shocks across markets:

$$\hat{D}^{X,dir}_{it't} \equiv \sum_{h\in H^{X}_{it'}} s^{sales}_{hit'} \tilde{\hat{M}}_{ht't}, \qquad \hat{S}^{I,dir}_{it't} \equiv \sum_{h\in H^{I}_{it'}} s^{mat}_{iht'} \tilde{\hat{X}}_{ht't} \tag{3.5}$$

where in the pre-period t', $H^{X}_{it'}$ ($H^{I}_{it'}$) is the set of markets in which firm i actively exports (imports) and $s^{sales}_{hit'}$ ($s^{mat}_{iht'}$) is the share of firm i's sales (material cost) accounted for by market h. Intuitively, a Chilean exporter with a high value of $\hat{D}^{X,dir}_{it't}$ relied more heavily in the past on exports to markets in which demand is now growing as measured by the idiosyncratic growth in these markets' imports from the rest of the world, $\tilde{\hat{M}}_{ht't}$, while a Chilean importer with a high value of $\hat{S}^{I,dir}_{it't}$ relied more heavily in the past on imports from markets in which production costs are now falling as measured by the idiosyncratic growth in these markets' exports to the rest of the world, $\tilde{\hat{X}}_{ht't}$.[8]

Furthermore, a Chilean firm may be affected by export demand and import cost shocks indirectly through domestic trade with other Chilean firms that directly export or import. To account for this, we construct a firm's *total* exposure to export demand and import cost shocks as follows:

$$\hat{D}^{X,tot}_{t't} \equiv \left[I - S^{sales}_{t'}\right]^{-1} \hat{D}^{X,dir}_{t't}, \qquad S^{I,tot}_{t't} \equiv \left[I - S^{mat}_{t'}\right]^{-1} \hat{S}^{I,dir}_{t't} \tag{3.6}$$

[8]Note that $\left\{\tilde{\hat{X}}_{ht't}, \tilde{\hat{M}}_{ht't}\right\}$ are parsed of aggregate shocks that may affect not only individual firms in Chile but also all firms in Chile that trade product $p(h)$ or that trade with country $c(h)$.

where $\hat{D}^{X,tot}_{t't}$ ($\hat{S}^{I,tot}_{t't}$) is a vector of total export demand (import cost) shocks with i^{th} element denoted by $D^{X,tot}_{it't}$ ($S^{I,tot}_{it't}$), $\hat{D}^{X,dir}_{t't}$ ($\hat{S}^{I,dir}_{t't}$) is a vector of the direct export demand (import cost) shocks with i^{th} element given by $D^{X,dir}_{it't}$ ($S^{I,dir}_{it't}$), and $\left\{S^{sales}_{t'}, S^{mat}_{t'}\right\}$ are the sales and material cost share matrices in the pre-period t' with (i,j) elements given respectively by the share of seller i's sales accounted for by customer j and the share of buyer i's material cost accounted for by supplier j. Naturally, the importance of each customer for a firm's total exposure to export demand shocks depends on sales shares, while the importance of each supplier for a firm's total exposure to import cost shocks depends on material cost shares. The Leontief inverses of the sales and material cost share matrices in equation (3.6) capture how connected each pair of Chilean firms are through downstream and upstream domestic trade linkages respectively, both directly and indirectly through other firms.[9] As we discuss below, our structural model motivates a similar definition of a firm's total exposure to demand and material cost shocks. Finally, we construct a firm's total exposure to final demand as:

$$s^{F,tot}_t \equiv \left[I - S^{sales}_t\right]^{-1} S^F_t \tag{3.7}$$

where $s^{F,tot}_t$ is a vector of total final demand exposure with i^{th} element denoted by $s^{F,tot}_{i,t}$ and S^F_t is a vector of final demand shares with i^{th} element equal to the share of firm i's sales accounted for by final sales.

With these shocks in hand, we estimate the following specification via OLS:

$$\hat{Y}_{it't} = \beta_0 + \beta_D \hat{D}^{X,tot}_{it't} + \beta_S \hat{S}^{I,tot}_{it't} + \beta_F s^F_{it'} + \delta_{\iota(i)} + \zeta_{it't} \tag{3.8}$$

where $\hat{Y}_{it't}$ is the log change in an outcome of interest for firm i, $\delta_{\iota(i)}$ is a fixed effect for the industry $\iota(i)$ of firm i, and $\zeta_{it't}$ is a residual. We estimate this in one long difference between the first and last years of our sample ($t' = 2005$ and $t = 2010$). Table 1 shows

[9]The Leontief inverse of the sales share matrix is the same as that used in Dhyne et al. (2022).

our estimates of the regression coefficients of interest $\{\beta_D, \beta_S\}$ for three different firm-level outcomes – sales, wage bill, and employment – where all standard errors are clustered at the 4-digit industry level.

There are four key observations. First, increases in customer demand and reductions in supplier cost have positive and significant effects on firm sales, wage bills, and employment. This provides evidence that supply chain linkages between firms are important for the transmission of demand and supply shocks. Second, workers do not fully capture the benefits of growth in firm sales, as the estimated coefficients in column 2 are smaller in magnitude than the corresponding coefficients in column 1. Third, workers capture more of the benefits of firm growth when such growth is driven by demand instead of supply shocks. Our estimates indicate that workers capture 0.89 cents (= 1.001/1.121) per dollar of sales growth following a positive demand shock but only 0.69 cents (= 0.237/0.341) per dollar of sales growth following a positive supply shock.[10] Finally, workers gain from firm growth through both the extensive margin (more employment) and intensive margin (higher wages), as the estimated coefficients in column 3 are smaller in magnitude than the corresponding coefficients in column 2.

4 A Model of Labor Markets and Production Networks

We now develop a structural model that is capable of rationalizing the stylized facts presented above. As emphasized in the introduction, the model adds to the empirical analysis presented thus far in two ways. First, it allows us to probe the underlying economic mechanisms that drive the stylized facts. Second, it allows us to quantify the importance of the production network for heterogeneity in firm earnings premia and labor shares through model-based counterfactuals. Note in particular that the measures of downstream and upstream access studied in section 3 are not exogenous primitives of a firm. Rather, like other

[10] We implement a test of the difference of the ratio of the estimates from Table 1 using the Delta method (and assuming that the covariance of the estimates is zero for simplicity). We find that the difference is statistically different from zero at the 10 percent level with a corresponding test statistic of 1.75.

Table 1: Estimates of pass-through from export demand and import cost shocks: baseline specification

	1. Sales	2. Wage bill	3. Employment
total demand shock	1.121	1.001	0.721
	(0.056)	(0.062)	(0.135)
total supply shock	0.341	0.237	0.196
	(0.038)	(0.014)	(0.024)
no. of observations	27,691	27,691	27,691

Notes: This table presents our estimates of the pass-through coefficients for export demand and import cost shocks in equation (3.8) for different outcome variables. All regressions are estimated via OLS using a single long difference between 2005 and 2010 with industry fixed effects. The sample used for this regression is a subset of the firm baseline sample discussed in Section 2. The sample size is smaller than the one from the firm baseline sample because we condition in this regression on firms that survive for 5 years between 2005 and 2010. Firms in this sample are larger in employment, value added, number of buyers and suppliers, and have a smaller labor share. Standard errors are shown in parentheses and are clustered at the 4-digit level. All outcome variables are measured in logs.

firm characteristics such as size, they depend endogenously on the set of production network linkages between firms. Hence, the reduced-form analysis in section 3 is not sufficient to attribute differences in earnings premia and labor shares to production network heterogeneity vis-à-vis other firm characteristics.

In this economy, there is a set of workers Ω^L and a set of firms Ω^F. Workers are heterogeneous in a characteristic that we refer to as ability, denoted by a, with an exogenous measure of each ability type denoted by $L(a)$ and the set of abilities denoted by A. Firms are also heterogeneous in a variety of characteristics that we specify below. Time is discrete and indexed by t.

4.1 Labor market

Firms and workers interact in the labor market as follows. Each firm i chooses wages $w_{it}(a)$ and offers exogenous amenities $g_i(a)$ for each worker of ability type a. In addition, workers derive idiosyncratic utility values ξ_{it} from employment at firm i, which are independent across workers and firms for a given t and follow a Gumbel distribution with cumulative distribution function $F_{\xi}(\xi_{it}) = e^{-e^{-\gamma\xi_{it}}}$, where the variance of the distribution is declining in the shape parameter γ.[11] Each worker observes the wage offers and amenities corresponding to her ability and chooses an employer to maximize utility. Workers are also residual claimants to firm profits, which are rebated through transfers in proportion to labor income, so that the rebate received by a worker earning wage w is equal to $\tau_t w$.[12] Total income is then used to finance final consumption, which is a CES aggregate of products produced by all firms in the economy with elasticity of substitution σ across products.

Formally, the potential utility of a worker of ability a with a vector $\xi_t \equiv \{\xi_{it}\}_{i\in\Omega^F}$ of idiosyncratic utility values is given by:

$$u_t(a|\,\xi_t) = \max_{i\in\Omega^F}\{\log(1+\tau_t)\,w_{it}(a) + \log g_i(a) + \xi_{it}\} \tag{4.1}$$

where we treat the price of the CES final consumption aggregate as the numeraire so that all income is in real terms. As is well known, under the Gumbel distribution of idiosyncratic utilities, the measure of workers of ability a that choose employment at firm i is given by:

$$L_{it}(a) = \kappa_{it}(a)\,w_{it}(a)^{\gamma} \tag{4.2}$$

where $\kappa_{it}(a)$ is a firm-specific labor supply shifter:

$$\kappa_{it}(a) \equiv L(a)\left[\frac{g_i(a)}{I_t(a)}\right]^{\gamma} \tag{4.3}$$

[11]It is simple to allow for correlation in ξ_{it} across firms by instead assuming that the vector $\{\xi_{it}\}_{i\in\Omega^F}$ has joint cumulative distribution function given by $exp\left[-\left(\sum_{i\in\Omega^F} e^{-\rho\gamma\xi_{it}}\right)^{\frac{1}{\rho}}\right]$, where the correlation of the distribution is increasing in the parameter $\rho \in [1,\infty)$. This version of the model is observationally equivalent to the version with independent draws of ξ_{it} across firms, with $\rho\gamma$ replacing γ.

[12]Rebating profits in proportion to labor income ensures that these transfers do not affect the sorting of workers across firms.

and $I_t(a)$ is a labor market index summarizing the wages and amenities offered by all firms for workers of ability a:

$$I_t(a) \equiv \left[\sum_{i \in \Omega^F} \left[g_i(a) \, w_{it}(a) \right]^{\gamma} \right]^{\frac{1}{\gamma}} \tag{4.4}$$

We assume that the cardinality of the set of firms Ω^F is large enough such that each firm views itself as atomistic in the labor market and hence takes the labor market indices $I_t(\cdot)$ as given when choosing wages. Each firm therefore behaves as though it faces an upward-sloping labor supply curve with a constant elasticity γ that is common to all firms and worker ability types. Intuitively, labor supply is more sensitive to differences in wages when there is less dispersion in preference shocks across firms.[13]

4.2 Production technology

Firm i produces output X_{it} using labor and materials as follows:

$$X_{it} = T_{it} \sum_{a \in A} F \left[\omega_{it} \phi_i(a) \, L_{it}(a) \, , M_{it}(a) \right] \tag{4.5}$$

where T_{it} is TFP, ω_{it} is labor productivity, $\phi_i(a)$ reflects worker-firm complementarities in production, and $M_{it}(a)$ is the quantity of materials assigned to workers of ability a.[14] The function F is a CES aggregator:

$$F(L, M) = \left(\lambda^{\frac{1}{\epsilon}} L^{\frac{\epsilon-1}{\epsilon}} + (1-\lambda)^{\frac{1}{\epsilon}} \, M^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \tag{4.6}$$

[13] Note that instead of arising from employer differentiation, labor market power could also stem from concentration (Chan et al. (2025), Jarosch et al. (2019)) or search frictions (Burdett and Mortensen (1998), Postel-Vinay and Robin (2002), Taber and Vejlin (2018)). Like ours, most of these models imply that wages are a markdown below the marginal revenue product of labor (MRPL) at a firm, where the firm earnings premium is the component of the MRPL that is common to all workers at a firm. Hence, the mechanisms linking the production network and worker earnings in our model are relevant for a broader class of models of the labor market.

[14] One can also think of certain types of capital inputs as sourced from suppliers in the production network under the label of "materials" if these inputs are chosen statically. Alternatively, it is straightforward to extend the production function to allow for a separate static capital input. See Online Appendix B for a formal discussion of this extension.

where λ controls the importance of labor relative to materials in production and ϵ is the elasticity of substitution between labor and materials.

While firms hire workers in the labor market by posting wages, materials are sourced through firm-to-firm trade in the production network. In particular, firm i produces a materials bundle by combining inputs from all of its suppliers $\Omega_{it}^{S} \subset \Omega^{F}$ using a CES technology, so that the total quantity of materials used in production $M_{it} \equiv \sum_{a \in A} M_{it}(a)$ satisfies:

$$M_{it} = \left[\sum_{j \in \Omega_{it}^{S}} \psi_{ijt}^{\frac{1}{\sigma}} \left(x_{ijt}\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{4.7}$$

where x_{ijt} denotes the quantity of inputs purchased by i from j and ψ_{ijt} is a relationship-specific productivity shifter. We assume that the latter can be decomposed as:

$$\psi_{ijt} = \psi_{it} \psi_{jt} \tilde{\psi}_{ijt} \tag{4.8}$$

where we refer to ψ_{it} as the *relationship capability* of firm i and $\tilde{\psi}_{ijt}$ as the relationship productivity residual. This decomposition allows firms to differ systematically in the productivity of their buyer-seller relationships. As is standard in the literature, we also assume the same elasticity of substitution σ across products in production as in final consumption, which simplifies the firm's profit maximization problem as it ensures that both final and intermediate demand have the same price elasticity.

We highlight several important features of the production technology. First, although it is straightforward to incorporate imperfect substitutability between workers of different abilities, the linear aggregation across worker types in equation (4.5) is necessary for the model to generate an earnings equation that is consistent with well-known statistical models of earnings such as those in Abowd et al. (1999) and Bonhomme et al. (2019). Second, although we allow for time-varying labor productivities ω_{it}, we restrict worker-firm complementarities $\phi_i(\cdot)$ to be time-invariant for identification purposes. Third, in the limit as

$\lambda \to 1$, output is produced using labor alone and the model simplifies to a version of the model studied in Lamadon et al. (2022). Finally, the production network is not restricted to be bipartite: firms can simultaneously be buyers and sellers, with $\Omega_{it}^{C} \equiv \left\{ j \in \Omega^{F} | i \in \Omega_{jt}^{S} \right\}$ denoting the set of customers for firm i. However, for tractability, we treat the set of active buyer-seller relationships in the economy as an exogenous primitive of the model and do not model network formation, an assumption that we discuss further below. Nonetheless, this imposes no restrictions on how the distribution of buyer-seller links is correlated with other firm primitives or how the network changes over time.

4.3 Price setting

Firms are monopolistically competitive in output markets, setting prices for their customers while taking the prices set by other firms as given. As with firm behavior in labor markets, we assume that firms behave atomistically in output markets and hence perceive a constant price elasticity of demand equal to $-\sigma$. Note that a firm's relationships with each of its customers are interlinked: a reduction in the price charged to one customer increases demand and hence raises both output and marginal cost, which in turn affects the choice of prices charged to other customers. Despite this, the profit-maximizing price charged by a firm to each of its customers (including final consumers) does not vary across customers:

$$p_{jit} = p_{it}, \forall j \in \Omega_{it}^{C} \cup \{F\} \tag{4.9}$$

The reason for this is simple. Each firm maximizes profits by choosing prices such that marginal revenue from each customer is equal to marginal cost. Since demand has a constant and common price elasticity of $-\sigma$, marginal revenue is proportional to price. Furthermore, even though marginal cost is increasing, it depends only on total output of the firm and hence is common across customers. Hence, prices do not vary across customers in equilibrium.

4.4 Firm network characteristics

The relevance of the production network for firm i's production decisions can be summarized by two sufficient statistics $\{D_{it}, S_{it}\}$ that we henceforth refer to as the *network characteristics* of the firm. These are given by:

$$D_{it} = E_t + \sum_{j \in \Omega_{it}^C} \Delta_{jt}\psi_{jit}, \qquad S_{it} = \left[\sum_{j \in \Omega_{it}^S} \Phi_{jt}\psi_{ijt}\right]^{\frac{1}{\sigma-1}} \tag{4.10}$$

The first term D_{it} is the firm's *demand shifter*, defined such that the firm's revenue given a choice of output price is equal to $D_{it}p_{it}^{1-\sigma}$, while the second term S_{it} is the firm's *supplier efficiency*, defined as the inverse of the unit cost of materials corresponding to the CES materials bundle in equation (4.7). Demand shifters depend on customer-specific intermediate demand shifters Δ_{jt} that we refer to as *buyer effects* (with aggregate household expenditure E_t serving as the buyer effect for final consumers), while supplier efficiencies depend on an inverse measure of supplier output prices Φ_{jt} that we refer to as *seller effects*.

The buyer and seller effects for firm i are in turn defined respectively as:

$$\Delta_{it} = E_{it}^M \left(S_{it}\right)^{1-\sigma}, \qquad \Phi_{it} \equiv p_{it}^{1-\sigma} \tag{4.11}$$

with $E_{it}^M \equiv M_{it}/S_{it}$ denoting total material cost. Intuitively, buyer effects are large when the buyer spends a lot on materials or has low supplier efficiency (so that competition for sales to the buyer from its full set of suppliers is low), while seller effects are large if the seller has a low output price. Firms that are connected to customers with large buyer effects then have high demand shifters, while those connected to suppliers with large seller effects have high supplier efficiencies.

4.5 Profit maximization and wage setting

The profit-maximization problem for firm i can now be expressed as:

$$\pi_{it} = \max_{\{w_{it}(a), M_{it}(a)\}_{a \in A}} \left\{ D_{it}^{\frac{1}{\sigma}} X_{it}^{\frac{\sigma-1}{\sigma}} - \sum_{a \in A} w_{it}(a) L_{it}(a) - \frac{1}{S_{it}} \sum_{a \in A} M_{it}(a) \right\} \tag{4.12}$$

where the maximization is subject to the labor supply curves (4.2) and production technology (4.5). Since the price of materials does not vary with worker ability, the marginal revenue product of materials must first of all be equalized across worker ability types in equilibrium. This is implied by the first-order condition with respect to $M_{it}(a)$:

$$\frac{1}{S_{it}} = \frac{1}{\mu} D_{it}^{\frac{1}{\sigma}} X_{it}^{-\frac{1}{\sigma}} T_{it} F_M(1, \nu_{it}) \tag{4.13}$$

where $\mu \equiv \frac{\sigma}{\sigma-1}$ is the output markup and $\nu_{it} \equiv \frac{M_{it}(a)}{\phi_i(a)\omega_{it}L_{it}(a)}$ is materials per efficiency unit of labor, which does not vary by worker ability.[15]

The first-order condition with respect to $w_{it}(a)$ then allows us to write:

$$w_{it}(a) = \eta \phi_i(a) W_{it} \tag{4.14}$$

where $\eta \equiv \frac{\gamma}{1+\gamma}$ is the wage markdown and W_{it} is the firm's *earnings premium*:

$$W_{it} = \frac{1}{\mu} D_{it}^{\frac{1}{\sigma}} X_{it}^{-\frac{1}{\sigma}} \omega_{it} T_{it} F_L(1, \nu_{it}) \tag{4.15}$$

As we show below, W_{it} differs from the firm earnings premium f_{it} in the reduced-form decomposition (3.1) only by a constant ($W_{it} = \eta f_{it}$). Equation (4.14) thus states the familiar result that wages are a constant markdown η over the marginal revenue product of labor (MRPL) of the respective worker types, $\phi_i(a) W_{it}$.

[15] In what follows, F_L and F_M denote the derivatives of F with respect to its first and second arguments, respectively.

Equilibrium output for firm i can then be characterized as follows:

$$X_{it} = T_{it} F\left(1, \nu_{it}\right) \bar{L}_{it} \tag{4.16}$$

$$\bar{L}_{it} = \left(\eta W_{it}\right)^{\gamma} \omega_{it} \bar{\phi}_{it} \tag{4.17}$$

$$\bar{\phi}_{it} \equiv \sum_{a \in A} \kappa_{it}\left(a\right) \phi_i\left(a\right)^{1+\gamma} \tag{4.18}$$

where $\bar{L}_{it} \equiv \sum_{a \in A} \phi_i\left(a\right) \omega_{it} L_{it}\left(a\right)$ is total efficiency units of labor hired by the firm and we define the $\bar{\phi}_{it}$ as the *sorting composite* for firm i, since this varies across firms only due to primitives that affect differential sorting of worker types across firms ($g_i\left(\cdot\right)$ and $\phi_i\left(\cdot\right)$). Expenditures on labor and materials can then be expressed respectively as:

$$E^L_{it} = \eta W_{it} \bar{L}_{it} / \omega_{it}, \qquad E^M_{it} = \nu_{it} \bar{L}_{it} / S_{it} \tag{4.19}$$

Given the firm's technological primitives, network characteristics, and sorting composite, equations (4.13), (4.15), (4.16), and (4.17) define a system of equations in the firm-level variables $\left\{W_{it}, \nu_{it}, X_{it}, \bar{L}_{it}\right\}$. In particular, the production network shapes earnings through the dependence of W_{it} on D_{it} and S_{it}.

4.6 General equilibrium

To close the model, it remains to characterize total consumer expenditure E_t. This is equivalent to aggregate value-added, where value-added at firm i, VA_{it}, is equal to the sum of labor costs and profits:

$$E_t = \sum_{i \in \Omega^F} \left(E^L_{it} + \pi_{it}\right) \tag{4.20}$$

We can then define the primitives and an equilibrium of the model as follows.

Definition **1** (model primitives)**.** *The primitives of the model at time t, Θ_t, are TFPs T_{it}, labor productivities ω_{it}, relationship capabilities ψ_{it}, relationship productivity residuals $\tilde{\psi}_{ijt}$, buyer-supplier linkages Ω^S_{it}, production complementarities $\phi_i\left(\cdot\right)$, amenities $g_i\left(\cdot\right)$, the worker* *ability distribution $L(\cdot)$, elasticities $\{\gamma, \epsilon, \sigma\}$, and the weight on labor in production λ.*

Definition **2** (equilibrium)**.** *Given a set of primitives, an equilibrium of the model at time t is a set of values for aggregate value-added E_t, network characteristics $\{D_{it}, S_{it}\}$, buyer and seller effects $\{\Delta_{it}, \Phi_{it}\}$, firm earnings premia W_{it}, output prices p_{it}, output X_{it}, labor efficiency units $\bar{L}_{it}$, sorting composites $\bar{\phi}_i$, input expenditures $\left\{E_{it}^L, E_{it}^M\right\}$, wages $w_{it}(\cdot)$, labor supply shifters $\kappa_{it}(\cdot)$, and labor market indices $I_t(\cdot)$, all of which satisfy equations (4.3), (4.4), (4.10), (4.11), (4.12), (4.13) (4.15), (4.16), (4.17), (4.18), (4.19), and (4.20).*

5 Equilibrium Analysis

We now use the model to shed light on the stylized facts presented in section 3 by providing a theoretical characterization of how the production network shapes worker earnings and labor shares of value-added and cost. This analysis will also provide context for the empirical results that follow.

5.1 Structural interpretation of reduced-form network access

We begin by providing a structural interpretation of the network access measures D_{it}^{net} and S_{it}^{net} presented in section 3. First, when firms choose the optimal quantities of inputs from their suppliers in order to minimize the cost of producing the materials bundle in equation (4.7), log sales between buyer i and seller j can be expressed as:

$$\log r_{ijt} = \log\left(\Delta_{it}\psi_{it}\right) + \log\left(\Phi_{jt}\psi_{jt}\right) + \log\tilde{\psi}_{ijt} \tag{5.1}$$

Comparing this with the reduced-form decomposition of firm-to-firm sales (3.2), we have $d_{it} \equiv \Delta_{it}\psi_{it}$, $s_{jt} \equiv \Phi_{jt}\psi_{jt}$, and $e_{ijt} \equiv \tilde{\psi}_{ijt}$. Therefore, in our model, the network access measures defined in equation (3.3) are given by:

$$D_{it}^{net} \equiv \sum_{j \in \Omega_{it}^{C}} \Delta_{jt} \psi_{jt} \tilde{\psi}_{jit}, \qquad S_{it}^{net} \equiv \sum_{j \in \Omega_{it}^{S}} \Phi_{jt} \psi_{jt} \tilde{\psi}_{ijt} \tag{5.2}$$

A firm's downstream access D_{it}^{net} is hence increasing in the buyer effects Δ_{jt} and relationship capabilities ψ_{jt} of its customers, while its upstream access S_{it}^{net} is increasing in the seller effects Φ_{jt} and relationship capabilities ψ_{jt} of its suppliers. Furthermore, from equation (4.10), we can express a firm's demand shifter and supplier efficiency respectively as:

$$D_{it} = E_t + \psi_{it} D_{it}^{net}, \qquad S_{it} = \left(\psi_{it} S_{it}^{net} \right)^{\frac{1}{\sigma - 1}} \tag{5.3}$$

Hence, the network access measures $\{D_{it}^{net}, S_{it}^{net}\}$ are sufficient statistics for how the production network shapes $\{D_{it}, S_{it}\}$ and therefore the firm earnings premium W_{it}.

5.2 The firm earnings premium

Recall from Fact 1 that firms with greater downstream and upstream access tend to have higher firm earnings premia. Although one cannot generally solve for W_{it} in closed-form, comparative static results can be used to highlight the mechanisms shaping differences in firm earnings premia.[16] To focus on the role played by the production network, we examine how shocks to demand shifters and supplier efficiencies in the network affect earnings premia.[17] In what follows, let $\hat{Y}_{it}$ denote the marginal log change in any variable Y_{it} and let $s_{it}^{M} \equiv \frac{E_{it}^{M}}{\frac{1}{\eta} E_{it}^{L} + E_{it}^{M}}$ denote the share of materials in firm i's production costs adjusted for wage markdowns. As before, let $\left\{ S_t^{sales}, S_t^{mat} \right\}$ denote the sales and material cost share matrices.

[16] When production technologies are of a Cobb-Douglas form ($\epsilon \to 1$), one can write the firm's profit maximization problem in terms of a value-added production function and obtain a closed-form solution for the firm earnings premium (see Online Appendix C). However, this value-added approach is valid *only* when $\epsilon \to 1$ and leads to two counterfactual predictions: it restricts labor cost shares to be constant across firms and, as we discuss in section 5.4 below, it implies complete pass-through of changes in firm value-added per worker to changes in worker earnings.

[17] See Appendix B.1 for a full discussion of comparative statics with respect to T_{it}, ω_{it}, and $\bar{\phi}_{it}$.

Now, consider vectors of shocks $\hat{D}_t \equiv \left\{\hat{D}_{it}\right\}_{i\in\Omega^F}$ and $\hat{S}_t \equiv \left\{\hat{S}_{it}\right\}_{i\in\Omega^F}$ to demand shifters and supplier efficiencies for all firms in the production network. The following Proposition summarizes how these shocks affect firm earning premia (and therefore worker earnings).[18] As we discuss in Appendix B.1, these results abstract from feedback effects arising from the fact that marginal costs are scale dependent due to upward-sloping labor supply curves, which are second-order and likely to be small empirically.

Proposition **1.** *The first-order effects of demand shifter and supplier efficiency shocks* $\left\{\hat{D}_t, \hat{S}_t\right\}$ *in the production network on firm i's earnings premium are given respectively by:*

$$\hat{W}_{it} = \Gamma_{it}\hat{D}_{it}^{total}, \qquad \hat{W}_{it} = (\sigma - \epsilon)\,\Gamma_{it} s_{it}^M \hat{S}_{it}^{total} \tag{5.4}$$

where $\Gamma_{it} \equiv \frac{1}{\gamma+\sigma\left(1-s_{it}^M\right)+\epsilon s_{it}^M} > 0$ *is the scale elasticity for firm i. The vectors of total demand and supplier efficiency shocks are defined as:*

$$\hat{D}_t^{total} \equiv \left[I - S_t^{sales}\delta_t^D\right]^{-1}\hat{D}_t, \qquad \hat{S}_t^{total} \equiv \left[I - S_t^{mat}\delta_t^U\right]^{-1}\hat{S}_t \tag{5.5}$$

where δ_t^D *and* δ_t^U *are diagonal matrices of downstream and upstream weights with* (i,i)*-elements given by* $\delta_{it}^D \equiv \frac{\gamma+\epsilon}{\gamma+\sigma\left(1-s_{it}^M\right)+\epsilon s_{it}^M}$ *and* $\delta_{it}^U \equiv \frac{(\gamma+\epsilon)s_{it}^M}{\gamma+\sigma\left(1-s_{it}^M\right)+\epsilon s_{it}^M}$*.*

The intuition for these results can be understood as follows. First, $\hat{D}_{it}^{total}$ and $\hat{S}_{it}^{total}$ are sufficient statistics summarizing a firm's total exposure to demand and supplier efficiency shocks in the production network. As discussed in section 3, the Leontief inverses of the sales and material cost share matrices account for both shocks that affect a firm directly as well as shocks that affect a firm indirectly through other firms in the production network. The main difference here is that our structural model implies that the sales and cost share matrices should be weighted by δ_t^D and δ_t^U, respectively. These weights arise from the fact that with upward-sloping labor supply curves, demand and supplier efficiency shocks are not

[18]Proofs of all propositions are provided in Appendix B.

passed through one-for-one into demand for inputs from suppliers and into output prices charged to customers, respectively.

Second, demand shocks have a positive effect on firm earnings premia ($\Gamma_{it} > 0$). This occurs because higher demand raises the output price of a firm, which translates into a higher MRPL and hence higher wages given the upward-sloping labor supply curves faced by each firm. The scale elasticity Γ_{it} summarizes the three conditions that are necessary for the existence of such scale effects: (i) the labor market is imperfectly competitive ($\gamma < \infty$), so that marginal costs of labor are increasing; (ii) the output market is imperfectly competitive ($\sigma < \infty$), so that higher demand raises output prices at the level of the firm; and (iii) labor and materials are imperfect substitutes ($\epsilon < \infty$), so that firms cannot fully escape from increasing marginal costs of labor by purchasing materials at constant marginal cost.

Third, the sign of the relationship between the firm earnings premium and supplier efficiency shocks depends on the sign of $\sigma - \epsilon$. This is because a change in supplier efficiency has both a scale and a substitution effect on W_{it}. On one hand, an increase in supplier efficiency is akin to a positive productivity shock, which induces the firm to expand in scale and hire more workers at higher wages. The strength of this scale effect is increasing in σ, since a firm's scale is more sensitive to changes in production costs when products are more differentiated. On the other hand, an increase in supplier efficiency also induces firms to change their relative usage of labor versus materials, which may decrease wages (if $\epsilon > 1$, so that labor and materials are substitutes) or increase wages (if $\epsilon < 1$, so that labor and materials are complements). Hence, the net effect depends on the relative magnitude of σ versus ϵ. In our estimation of the model's parameters below, we find that $\sigma > \epsilon$ and hence greater supplier efficiencies induce higher wages.

Note that even though Proposition 1 is presented in terms of changes over time, it can equivalently be interpreted as characterizing how firms with different values of demand shifters and supplier efficiencies will have different earnings premia. Hence, firms with greater demand shifters D_{it} and supplier efficiencies S_{it} will pay higher wages, which is consistent with Fact 1. Similarly, increases in demand for a firm or its customers or increases in supplier efficiencies for a firm or its suppliers lead to higher wages, which is consistent with Fact 3. Note also that demand and supply shocks of the same magnitude can have potentially different effects on earnings, a point that we will examine in more detail below.

5.3 Labor shares of cost and value-added

Recall from Fact 2 that firms with greater downstream and upstream access in the production network tend to have lower labor shares of both cost and value-added. To see how these patterns might arise in our model, first note that a firm's labor share of cost, $s_{it}^{L/C} \equiv \frac{E_{it}^L}{E_{it}^L + E_{it}^M}$, is completely determined by its productivity-adjusted price of labor relative to materials:

$$s_{it}^{L/C} = 1 - \left[1 + \eta \left(\frac{\lambda}{1-\lambda}\right) \left(\frac{W_{it} S_{it}}{\omega_{it}}\right)^{1-\epsilon}\right]^{-1} \tag{5.6}$$

Furthermore, with constant output markups μ and constant labor markdowns η, a firm's labor share of value-added, $s_{it}^{L/VA} \equiv \frac{E_{it}^L}{VA_{it}}$, is uniquely determined by its labor cost share:

$$s_{it}^{L/VA} = \frac{\eta s_{it}^{L/C}}{\mu s_{it}^{L/C} + (\mu - 1)\, \eta \left(1 - s_{it}^{L/C}\right)} \tag{5.7}$$

Evidently, $s_{it}^{L/VA}$ is increasing in $s_{it}^{L/C}$: as a matter of accounting, a firm's labor share of value-added tends to be greater when labor makes up a larger share of its production cost.

Now recall from Proposition 1 that increases in the demand shifter D_{it} raise W_{it}. Hence, greater downstream access leads to an increase in the relative price of labor to materials, $W_{it} S_{it}$. On the other hand, increases in supplier efficiency S_{it} have both a direct effect on this relative price and an indirect effect that operates through W_{it}. As we show in the proof of the following Proposition, the net of these two effects is such that $W_{it} S_{it}$ is strictly increasing in S_{it}. Hence, equations (5.6) and (5.7) allow us to summarize the relationship between a firm's labor shares, demand shifter D_{it}, and supplier efficiency S_{it} as follows.

Proposition 2. *A firm's labor share of cost* $s_{it}^{L/C}$ *and labor share of value-added* $s_{it}^{L/VA}$ *are: (i) strictly decreasing in* D_{it} *and* S_{it} *if* $\epsilon > 1$*; (ii) strictly increasing in* D_{it} *and* S_{it} *if* $\epsilon < 1$*; and (iii) independent of* D_{it} *and* S_{it} *if* $\epsilon = 1$.

As discussed below, we estimate that $\epsilon > 1$, so that firms with greater downstream and upstream network access tend to have lower labor shares of both cost and value-added, which is consistent with Fact 2.

5.4 Worker earnings, firm size, and growth

Recall from Fact 3 that workers do not fully capture the benefits of firm growth arising from increases in customer demand or reductions in supplier input costs. In particular, earnings increase less than value-added while average wages increase less than value-added per worker. Furthermore, demand shocks benefit workers more than cost shocks conditional on the same growth in firm size. Our model rationalizes these patterns as follows.

First, we can express a firm's earnings premium in terms of its size (sales or value-added), labor share (of cost or value-added), and sorting composite:[19]

$$\log W_{it} = \text{const.} + \frac{1}{1+\gamma}\log R_{it} + \frac{1}{1+\gamma}\log s_{it}^{L/C} - \frac{1}{1+\gamma}\log \bar{\phi}_{it} \tag{5.8}$$

$$= \text{const.} + \frac{1}{1+\gamma}\log VA_{it} + \frac{1}{1+\gamma}\log s_{it}^{L/VA} - \frac{1}{1+\gamma}\log \bar{\phi}_{it} \tag{5.9}$$

This makes it clear that firm size is generally not a sufficient statistic for the firm earnings premium. Even if one accounts for differences in sorting through $\bar{\phi}_{it}$ (as, for example, in Lamadon et al. (2022)), heterogeneity in the production network still contributes to differences in earning premia conditional on firm size through differences in labor shares (as long as $\epsilon \neq 1$ so that labor shares do in fact vary across firms). This further implies that unpack-

[19]To derive equation (5.8), note that with constant markups, total input costs are directly proportional to sales. Hence, $R_{it}s_{it}^{L/C}$ is directly proportional to the wage bill E_{it}^{L}. Using the expression for the wage bill in equation (4.19), taking logs, and rearranging terms gives equation (5.8). Equation (5.9) is derived in a similar manner, since $VA_{it}s_{it}^{L/VA}$ is equal to the wage bill.

ing the determinants of heterogeneity in firm size (as in Bernard et al. (2022), for example) is complementary but not equivalent to unpacking the determinants of heterogeneity in worker earnings.

Equations (5.8) and (5.9) also highlight why workers may not fully capture the benefits of growth in firm size: shocks that increase sales or value-added may also lead to a reduction in a firm's labor shares. To see this more clearly, consider a shock to a firm's demand shifter D_{it} or supplier efficiency S_{it}. Let:

$$\beta_D^{EL/VA} \equiv \frac{\partial \log E_{it}^L}{\partial \log D_{it}} / \frac{\partial \log VA_{it}}{\partial \log D_{it}} \qquad \beta_S^{EL/VA} \equiv \frac{\partial \log E_{it}^L}{\partial \log S_{it}} / \frac{\partial \log VA_{it}}{\partial \log S_{it}} \tag{5.10}$$

denote the marginal increase in worker earnings relative to the marginal increase in firm value-added following a demand and supplier efficiency shock, respectively. Similarly, let $\beta_D^{W/VAPW}$ and $\beta_S^{W/VAPW}$ denote the same pass-through coefficients, but for changes in the firm earnings premium W_{it} relative to value-added per worker $VAPW_{it} \equiv VA_{it}/L_{it}$. The following Proposition shows that these pass-through coefficients depend critically on the labor-materials substitution elasticity, ϵ.

Proposition **3.** *Suppose $\sigma > \epsilon$ so that greater supplier efficiencies raise firm earnings premia. Then the pass-through coefficients $\beta_D^{EL/VA}$, $\beta_S^{EL/VA}$, $\beta_D^{W/VAPW}$, and $\beta_S^{W/VAPW}$ are: (i) strictly less than one if $\epsilon > 1$; (ii) strictly greater than one if $\epsilon < 1$; and (iii) equal to one if $\epsilon = 1$. In addition, the relative pass-through coefficients for demand versus supplier efficiency shocks, $\beta_D^{EL/VA}/\beta_S^{EL/VA}$ and $\beta_D^{W/VAPW}/\beta_S^{W/VAPW}$, are: (i) strictly greater than one if $\epsilon > 1$; (ii) strictly less than one if $\epsilon < 1$; and (iii) equal to one if $\epsilon = 1$.*

The intuition for these results is as follows. First, when labor and materials are substitutes ($\epsilon > 1$), increases in demand and supplier efficiency not only raise a firm's value-added and VAPW but also lead the firm to substitute away from labor toward materials (see the discussion of Proposition 2). This substitution effect partially offsets the increase in wages that arises from greater demand or supplier efficiency, as the firm grows in size but relies relatively more on materials for production compared with labor. Furthermore, supplier efficiency shocks directly affect the relative price of labor to materials, $W_{it}S_{it}$, whereas demand shocks only have an indirect effect on this relative price through the scale-dependence of the firm earnings premium. Hence, the offsetting substitution effect is stronger with supplier efficiency shocks compared with demand shocks, which explains why workers benefit relatively more from the latter compared with the former conditional on the same growth in firm size. When $\epsilon < 1$, labor and materials are complements instead of substitutes, so the converse effects obtain. Finally, when $\epsilon = 1$, labor cost shares are constant and hence both demand and supplier efficiency shocks raise earnings (firm premia) in direct proportion to firm value-added (VAPW).

Importantly, Proposition 3 highlights that a necessary condition for our model to rationalize the outcomes in Fact 3 is that labor and materials are substitutes ($\epsilon > 1$). This is precisely what we find below when we estimate ϵ using the Chilean data. In particular, imposing Cobb-Douglas technology (or assuming a value-added production function) implies complete pass-through of changes in value-added (VAPW) to changes in worker earnings (firm earnings premia) regardless of the underlying shock, which is strongly rejected by both our findings and the empirical literature more broadly (as in Berger et al. (2022) and Kline et al. (2019), for example). For a detailed characterization of how pass-through depends on ϵ in our model, see Online Appendix D.

5.5 Discussion of key model features

We briefly summarize here the key features of our model underpinning the theoretical propositions presented above and their connection to our empirical findings, as well as how the framework that we have chosen to develop compares with other models of firms and production networks. It should first of all be evident that for demand and supply shocks to affect wages at the firm level (Fact 3 and Proposition 1), firms must face upward-sloping labor supply curves. Furthermore, in order for positive demand and supply shocks to lead to lower labor shares of cost and value-added (Fact 2 and Proposition 2), and for workers to benefit more from firm-level demand shocks than supply shocks (Fact 3 and Proposition 3), it is necessary for labor and materials to be substitutes ($\varepsilon > 1$).

These two key features of our model also motivate other points of departure from existing models of labor markets and production networks. The assumption of upward-sloping labor supply curves implies that firms face marginal production costs that are increasing with output, whereas many existing models of endogenous production network formation (such as those in Huneeus (2019), Lim (2019), and Bernard et al. (2022)) impose the restriction that marginal production costs are independent of output. This is needed for tractability because in these frameworks, endogeneity of the network is modeled by assuming that firms pay a fixed cost to sell to each customer, and with constant marginal production costs, the decision of a firm to sell to one customer can be analyzed independently of who else the firm sells to. Since our focus is on labor market outcomes at the firm level, we choose to model upward-sloping labor supply curves at the expense of endogenous network formation, as in Dhyne et al. (2022). An alternative approach would be to adopt the search and matching model of endogenous network formation in Demir et al. (2024); however, the tractability of this framework requires the assumption of Cobb-Douglas production functions, the departure from which is crucial to our analysis as discussed above.[20]

Finally, recent work by Dhyne et al. (2022) has emphasized the role of fixed labor costs for explaining the fact that when a firm grows in size, input purchases tend to grow more rapidly than labor cost. It is clear from part (i) of Proposition 2 that if a firm grows due to positive demand or supply shocks (higher D_{it} or S_{it}), the firm's labor share of cost will decline as long as labor and materials are substitutes ($\varepsilon > 1$), which in turn implies that if total cost is growing, input purchases must grow faster than labor cost. Hence, a corollary of

[20] Demir et al. (2024) emphasize positive assortative matching between firms on product quality. While we do not explicitly have product quality in our model, firms differ in multiple primitives (TFP, labor-augmenting productivity, relationship capability, production complementarities, worker amenities), and we calibrate the production network in our model below to match the observed pattern of buyer-seller linkages between firms with different values of these primitives.

Proposition 2 is that fixed labor costs are not strictly necessary to rationalize the observation that input purchases tend to grow more than labor cost when firms increase in size, as long as labor and materials are substitutes. Furthermore, in Appendix C, we show that fixed labor costs are neither necessary nor sufficient for explaining our key finding that workers benefit more from firm-level demand shocks than supply shocks (Fact 3 and Proposition 3). In essence, a necessary condition to explain such asymmetric passthrough is that labor and materials are substitutes, as explained following Proposition 3, regardless of whether there are fixed labor costs or not. For these reasons, we have chosen to abstract from fixed labor costs in our framework.

6 Connecting the Model to Data

We now turn towards identification and estimation of the model's parameters. We first describe additional assumptions that are helpful for identification (section 6.1) and then present our identification strategy and estimation results (section 6.2).

6.1 Assumptions for identification

Assumptions required for identification relate mainly to functional forms and the underlying stochastic processes for time-varying primitives. As we move toward connecting the model with worker-level data, we now also explicitly index individual workers by m.

Assumption **6.1.** *The ability of worker m at time t, a_{mt}, is comprised of a permanent component $\bar{a}_m$ and a time-varying component $\hat{a}_{mt}$, where $\log \hat{a}_{mt}$ follows a stationary mean-zero stochastic process that is independent of $\bar{a}_m$.*

This distinction between permanent and transient worker ability will be important for a decomposition of worker earnings into firm and worker effects that we implement below.

Assumption **6.2.** *Worker-firm production complementarity takes the following form:*

$$\log \phi_i (a_{mt}) = \theta_i \log \bar{a}_m + \log \hat{a}_{mt} \tag{6.1}$$

and the firm amenity function depends only on permanent worker ability, $g_i (a_{mt}) = g_i (\bar{a}_m)$*.*

This assumption restricts the two sources of worker-firm interactions in the model – production complementarities and amenities – to be time-invariant. This will be important for identification, as time-varying worker-firm complementarities cannot generally be identified. In what follows, we refer to the primitive θ_i simply as the *production complementarity* of firm i.

Assumption **6.3.** *Relationship productivity residuals* $\tilde{\psi}_{ijt}$ *are iid across firm pairs and time.*

This assumption will be important for the decomposition of firm-to-firm transactions into buyer and seller effects that we implement below.

Assumption **6.4.** *Time-varying firm primitives* $\{T_{it}, \omega_{it}, \psi_{it}\}$ *follow stationary first-order Markov processes with innovations that are iid across both firms and time.*

This follows well-known papers in the literature on production function estimation such as Olley and Pakes (1996) and Doraszelski and Jaumandreu (2018). As described below, we adopt the approach in the latter paper to estimate parameters of the production function and hence consider this Markov structure.

Assumption **6.5.** *The stochastic processes for transient worker ability* $\hat{a}_{mt}$*, time-varying firm primitives* $\{T_{it}, \omega_{it}, \psi_{it}\}$*, and relationship productivity residuals* $\tilde{\psi}_{ijt}$ *are mutually independent.*

Independence of the stochastic processes for worker and firm characteristics ensures that residual worker earnings due to transient ability shocks are uncorrelated with the characteristics of the worker's firm and is the same as the orthogonality assumption imposed in

Lamadon et al. (2022). Furthermore, independence of firm primitives and relationship productivity residuals does not imply that firms match at random, only that they do not match based on $\tilde{\psi}_{ijt}$.

6.2 Identification strategy and estimation results

6.2.1 Labor supply elasticity

With Assumption 6.2, the log wage of worker m at firm i and time t is:

$$\log w_{imt} = \theta_i \log \bar{a}_m + \log \eta W_{it} + \log \hat{a}_{mt} \tag{6.2}$$

which is consistent with the decomposition of log earnings in equation (3.1) with $x_m \equiv \log \bar{a}_m$ and $f_{it} \equiv \eta W_{it}$. Using equations (4.17) and (4.19) to substitute for W_{it} in terms of the wage bill E^L_{it}, we obtain:

$$\log w_{imt} = \theta_i \log \bar{a}_m + \frac{1}{1+\gamma} \log E^L_{it} - \frac{1}{1+\gamma} \log \bar{\phi}_{it} + \log \hat{a}_{mt} \tag{6.3}$$

Note that the sorting composite $\bar{\phi}_{it}$ is time-varying only through general equilibrium terms (the labor market indices I_t). Since Assumptions 6.1 and 6.4 impose stationarity on the distributions of time-varying worker and firm primitives respectively, we treat I_t and hence $\bar{\phi}_{it}$ as time-invariant. Restricting attention to workers that do not change employers between t and $t+1$ (stayers) and taking first-differences of equation (6.3) then gives:

$$\Delta \log w_{imt} = \frac{1}{1+\gamma} \Delta \log E^L_{it} + \Delta \log \hat{a}_{mt} \tag{6.4}$$

Intuitively, the change in a firm's wage bill is a sufficient statistic for all firm-level shocks that matter for changes in the earnings of stayers at the firm, including shocks to a firm's customers and suppliers in the production network. Since the labor supply elasticity γ controls the extent of imperfect competition in the labor market and mediates the extent of rent-sharing between a firm and its employees, the pass-through of changes in wage bills to changes in wages is informative about the magnitude of γ. In particular, stronger pass-through implies greater labor market power and a smaller value of γ.

This approach resembles the pass-through analysis in Guiso et al. (2005) and Lamadon et al. (2022), except that these papers consider changes in firm value-added whereas our model motivates using changes in wage bills instead. This distinction is moot if output markets are perfectly competitive ($\sigma \to \infty$) or if intermediates are not used in production ($\lambda \to 1$), since the wage bill is then a constant fraction of value-added for every firm. In general, however, wage bills are not proportional to value-added and identification of γ requires leveraging changes in the former instead of the latter. Taking first-differences of equation (5.9) also shows that a firm's labor share enters as a residual into the relationship between worker earnings and firm value-added. Since typical instruments for value-added are also likely to be correlated with labor shares, this cautions against IV strategies to identify γ from the pass-through of firm value-added shocks into worker earnings.

To estimate γ in practice, we first remove age and year effects from log wages (since these are outside of our model) by regressing the latter on a vector of year dummy variables and a cubic polynomial in worker age, then treating the residual as our measure of $\log w_{imt}$.[21] We then estimate the pass-through elasticity in equation (6.4) using the stayers sample. An IV approach that instruments $\Delta \log E^L_{it}$ with its own lags of at least 3 and greater is robust to allowing for measurement error in observed log wage bills of an MA(1) form, whereas OLS estimation of equation (6.4) is not.[22] Hence, in our preferred specification, we instrument the change in the log wage bill using a cubic polynomial in 3, 4 and 5 of its own lags (stopping at 5 lags due to sample size considerations). The results obtained from this specification are shown in Column 1 of Table 2. We estimate a pass-through elasticity of around 0.16, which implies a labor supply elasticity of $\gamma = 5.5$.

For comparison, we also report results obtained from other specifications. In Column

[21] We obtain the year of birth for each individual who is alive in 2018 from a civil registry database.
[22] See Online Appendix E.1 for a formal discussion.

2, we instrument $\Delta \log E^L_{it}$ using only a cubic polynomial in its third lag. In this case, the pass-through elasticity increases to 0.18 ($\gamma = 4.6$), although the first-stage F-statistic for this specification is substantially smaller than the corresponding F-statistic in our preferred specification. Nonetheless, both the estimates reported in Columns 1 and 2 are in line with estimates of pass-through elasticities reported in the literature.[23] In Column 3, we show the OLS estimate that ignores potential measurement error in wage bills. We find that the pass-through elasticity is substantially larger at 0.27. This implies $\gamma = 2.7$, which is half of our preferred estimate.[24]

Table 2: Estimation of labor supply elasticity (γ)

	$\Delta \log w_{imt}$		
	(1)	(2)	(3)
$\Delta \log E^L_{it}$	0.155	0.177	0.268
	(0.006)	(0.007)	(0.001)
γ	5.5	4.6	2.7
Strategy	GMM	GMM	OLS
Instruments Accumulated Lags	5	3	
First Stage F-Stat	2325	1426	
Number of Observations	2,507,868	2,507,868	2,507,868

Notes: This table presents results from the pass-through regression based on equation (6.4). All GMM strategies use different instruments of cubic polynomials of lags of wage bill and is implemented in two stages with a robust weighting matrix used to compute standard errors. Column 1 (our preferred specification) uses changes of wage bill lagged for 3, 4 and 5 periods as instruments. Column 2 uses changes of wage bill lagged for 3 periods as instruments. Column 3 estimates the model with OLS, which ignores measurement error on the wage bill. The sample used for the analysis of this table is a subset of the stayers sample described in Section 2. It is a subset given that we take first differences over time and use several variables in lags. Standard errors are shown in parentheses.

[23] For example, in a survey, Card et al. (2018) report values for this elasticity between 0.10 and 0.15. Lamadon et al. (2022) in particular estimate a pass-through elasticity of 0.15.

[24] For additional robustness, we also consider a difference-in-difference estimator for γ proposed by Lamadon et al. (2022), which considers firms with above-median values of $\Delta \log E^L_{it}$ as treated and others as untreated. We find estimates of γ that are similar to our preferred estimate using this approach, the details of which are relegated to Online Appendix E.1 for brevity.

6.2.2 Worker abilities and firm production complementarities

We identify worker abilities $\{\bar{a}_m, \hat{a}_{mt}\}$ and firm production complementarities θ_i using the decomposition of log worker earnings studied by Bonhomme et al. (2019). We first move all time variation in wage bills to the left-hand side of the earnings equation (6.3):

$$\log \tilde{w}_{imt} = \underbrace{\theta_i \log \bar{a}_m}_{\text{worker-firm interaction}} + \underbrace{\log \bar{W}_i}_{\text{firm FE}} + \underbrace{\log \hat{a}_{mt}}_{\text{residual}} \tag{6.5}$$

where $\bar{W}_i \equiv \left(\bar{E}_i^L / \bar{\phi}_i\right)^{\frac{1}{1+\gamma}}$ is a time-invariant firm effect, $\log \bar{E}_i^L$ is the time-average of firm i's log wage bill, and $\log \tilde{w}_{imt} \equiv \log w_{imt} - \frac{1}{1+\gamma}\left(\log E_{it}^L - \log \bar{E}_i^L\right)$ is log worker earnings residualized by the innovation in its employer's log wage bill. Given the orthogonality of $\hat{a}_{mt}$ to both $\bar{a}_m$ (Assumption 6.1) and employer primitives (Assumptions 6.2 and 6.5), we then obtain the key identifying restriction in Bonhomme et al. (2019):

$$\mathbb{E}\left[\frac{1}{\theta_j}\left(\log \tilde{w}_{jm,t+1} - \log \bar{W}_j\right) - \frac{1}{\theta_i}\left(\log \tilde{w}_{im,t} - \log \bar{W}_i\right) | m \in M_{t,t+1}^{i \to j}\right] = 0 \tag{6.6}$$

where the expectation is taken over the set of workers $M_{t,t+1}^{i \to j}$ that move from firm i at time t to firm j at time $t+1$. In principle, this restriction gives $\left|\Omega^F\right|^2$ moment conditions for identification of $2\left|\Omega^F\right|$ parameters (θ_i and $\bar{W}_i$ for every firm), where intuitively, changes in earnings accompanying changes in employers are informative about the firm-specific determinants of earnings.

In practice, we follow Bonhomme et al. (2019) and first assign each firm in our data to one of ten clusters via a K-means clustering algorithm that targets moments of the within-firm distribution of residualized earnings $\tilde{w}_{imt}$, with $k(i)$ denoting the *earnings cluster* of firm i.[25] Although not strictly necessary for identification, this reduces the dimension of the parameter set that needs to be estimated and ameliorates the well-known limited mobility bias issue. We then estimate $\left\{\bar{W}_{k(i)}, \theta_{k(i)}\right\}$ via limited information maximum likelihood using

[25] Online Appendix E.2 provides more details including diagnostics of the clustering procedure and robustness of our results with respect to the number of clusters.

the movers sample and the moment condition (6.6).[26] Permanent worker ability is then recovered as $\log \bar{a}_m = \mathbb{E}\left[\frac{\log \tilde{w}_{imt} - \log \bar{W}_{k(i)}}{\theta_{k(i)}}\right]$, while transient worker ability is recovered as the residual in earnings given our estimates of all other determinants of earnings. Furthermore, the time-varying firm earnings premium W_{it} can be recovered as $\log W_{it} = -\log \eta + \log \bar{W}_{k(i)} + \frac{1}{1+\gamma}\left(\log E^L_{it} - \log \bar{E}^L_i\right)$, which is firm-specific even though $\bar{W}_{k(i)}$ is restricted to vary only by cluster. Note that this approach allows us to estimate the decomposition of worker earnings (3.1) presented in section 3.

Our estimates imply a positive correlation between $\log \bar{W}_k$ and θ_k, indicating that firms with higher wage premia are also those where workers of higher ability are more productive.[27] In addition, the estimates that we obtain for θ_k are indicative of strong production complementarities. For example, they imply that workers in the top 2% of the permanent ability distribution are around 40% more productive when employed at firms in the highest $\bar{W}_k$ cluster than at firms in the lowest $\bar{W}_k$ cluster.[28]

6.2.3 Relationship capabilities and productivity residuals

Rewriting the firm-to-firm sales equation (5.1), we have:

$$\log r_{ijt} = \log \tilde{\Delta}_{it} + \log \tilde{\Phi}_{jt} + \log \tilde{\psi}_{ijt} \tag{6.7}$$

where $\tilde{\Delta}_{it} \equiv \Delta_{it}\psi_{it}$ and $\tilde{\Phi}_{jt} \equiv \Phi_{jt}\psi_{jt}$. Under Assumption 6.3, $\tilde{\Delta}_{it}$ is identified from purchases by firm i from all its suppliers controlling for total sales by these suppliers, $\tilde{\Phi}_{jt}$ is identified from sales by firm j to all its customers controlling for total expenditures by these customers, and $\tilde{\psi}_{ijt}$ is identified from the residual.[29] In practice, we estimate the terms on the right-hand side of equation (6.7) by regressing log firm-to-firm transactions on buyer-year and seller-year

[26] We thank Bradley Setzler for providing the code for this step of the estimation procedure.

[27] This positive correlation is also documented in Lamadon et al. (2022) using US data.

[28] Further details of these findings are relegated to Online Appendix E.2 for brevity.

[29] Since matching in intermediate input markets can occur many-to-many (each seller can have several buyers at once and each buyer can have several sellers), this identification strategy only requires cross-sectional moments. This is in contrast with identification of the worker and firm earnings effects in equation (6.5), which requires movements of workers across firms over time.

fixed effects.[30] It is important to emphasize here that this approach allows relationships to be selected based on the buyer or seller characteristics $\left\{\tilde{\Delta}_{it}, \tilde{\Phi}_{jt}\right\}$, but not on the relationship-specific characteristic $\tilde{\psi}_{ijt}$, which is the same assumption and approach to decomposing firm-to-firm sales used in Bernard et al. (2022).

To separately identify buyer effects Δ_{it}, seller effects Φ_{it}, and relationship capabilities ψ_{it} from $\tilde{\Delta}_{it}$ and $\tilde{\Phi}_{it}$, first note that the share of a firm's total sales that come from the network (i.e. excluding final sales) can be expressed as $s_{it}^{net} = \frac{\psi_{it} \sum_{j\in\Omega_{it}^C} \tilde{\Delta}_{jt}\tilde{\psi}_{jit}}{E_t + \psi_{it} \sum_{j\in\Omega_{it}^C} \tilde{\Delta}_{jt}\tilde{\psi}_{jit}}$. Solving for ψ_{it}, we obtain:

$$\psi_{it} = E_t \left(\frac{s_{it}^{net}}{1 - s_{it}^{net}}\right) \frac{1}{\sum_{j\in\Omega_{it}^C} \tilde{\Delta}_{jt}\tilde{\psi}_{jit}} \tag{6.8}$$

which allows identification of ψ_{it} up to a normalizing constant given observable network sales shares s_{it}^{net}.[31] Intuitively, a higher value of ψ_{it} increases sales within the network but not to final consumers. Buyer and seller effects are then easily recovered from $\tilde{\Delta}_{it}$ and $\tilde{\Phi}_{it}$.

6.2.4 Product substitution elasticity

It is straightforward to show that the ratio of a firm's sales to its production costs adjusted for wage markdowns is equal to the standard CES markup, so that $\frac{R_{it}}{\frac{1}{\eta}E_{it}^L + E_{it}^M} = \frac{\sigma}{\sigma-1}$. Solving for σ then gives:

$$\sigma = \frac{R_{it}}{R_{it} - \frac{1}{\eta}E_{it}^L - E_{it}^M} \tag{6.9}$$

where the denominator is profits adjusted for wage markdowns. Intuitively, σ controls the extent of output market power and hence is identified from the ratio of firm sales to profits.

Since we do not allow σ to vary across firms in the model, we estimate σ in practice using the ratio of the sample averages of the numerator and denominator on the right-hand side of equation (6.9). This gives an estimate of $\sigma = 3.1$ for the average year in our sample.

[30] Technical details of the implementation are provided in Online Appendix E.4.

[31] The normalizing constant for ψ_{it} is irrelevant for the same reason that one can normalize either Hicks neutral productivity or one factor-biased productivity without loss of generality.

6.2.5 Labor-materials substitution elasticity and labor productivities

Given the first-order Markov structure of firm productivity primitives in Assumption 6.4, we can first express log labor productivity as $\log \omega_{it} = F^{\omega}(\log \omega_{i,t-1}) + \xi_{it}^{\omega}$, where F^{ω} is a Markov transition function and ξ_{it}^{ω} is an innovation. Combining equations (4.13), (4.15), and (4.19), we then obtain:

$$\log \frac{E_{it}^{M}}{E_{it}^{L}} = \log \left[\frac{1}{\eta} \left(\frac{1-\lambda}{\lambda} \right) \right] + (\epsilon - 1) \log \frac{W_{it}}{Z_{it}} + (1-\epsilon) F^{\omega}(\log \omega_{i,t-1}) + (1-\epsilon) \xi_{it}^{\omega} \qquad (6.10)$$

where recall that $Z_{it} \equiv S_{it}^{-1}$ is the unit cost of materials. This is the standard relationship between relative factor expenditures ($\frac{E_{it}^{M}}{E_{it}^{L}}$) and relative factor prices ($\frac{W_{it}}{Z_{it}}$) implied by cost minimization under CES technologies. Note, however, that W_{it} and Z_{it} are not simple averages of the heterogeneous wages and material prices that a firm pays to its workers and suppliers. Instead, W_{it} is identified from the decomposition of worker earnings described in section 6.2.2, while Z_{it} can be constructed from the seller effects and relationship productivities obtained from the firm-to-firm sales decomposition discussed in section 6.2.3 (given an estimate of σ). This highlights the importance of merged employer-employee and firm-to-firm data for identifying these factor price aggregates within the firm.

Under Assumption 6.4, identification of the labor-materials substitution elasticity ϵ from equation (6.10) then follows the strategy in Doraszelski and Jaumandreu (2018). We implement this on the baseline firm-level dataset, using polynomials in one-period lagged factor prices and expenditures to instrument for $\log \frac{W_{it}}{Z_{it}}$, as well as a cubic polynomial control function in $\log \frac{E_{it-1}^{M}}{E_{it-1}^{L}}$ and $\log \frac{W_{it-1}}{Z_{it-1}}$ to control for $F^{\omega}(\log \omega_{i,t-1})$, with a detailed derivation of the approach relegated to Appendix E.5. Since there are many potential instruments available, we implement estimation using all possible combinations of the instruments and vary the order of the polynomials used. Among specifications that deliver a p-value of the Hansen J test above 0.1, we then choose the specification that yields the highest F-statistic.

Table 3 presents our results.[32] Our preferred specification based on the criteria above is shown in Column 1. This uses quadratic polynomials in $\{E^M_{it-1}, E^L_{it-1}\}$ as instruments and delivers an estimate of $\epsilon = 1.5$, implying that labor and materials are substitutes ($\epsilon > 1$).[33] For comparison, we also present results from other specifications. In Column 2, we use estimates of W_{it} based on the wage model and estimation strategy in Abowd et al. (1999), which rules out worker-firm interactions in equation (6.5) and does not cluster firms to deal with limited mobility. Applying the instrument selection criteria above, we use a linear polynomial in $\{E^M_{it-1}, E^L_{it-1}, W_{it-1}, Z_{it-1}\}$ as instruments and find $\epsilon = 1.6$, which is not statistically different from our preferred estimate in Column 1. In Column 3, we follow the standard approach in the literature and use average firm wages in place of W_{it}. Our instrument set in this case is comprised of quadratic polynomials in $\{W_{it-1}, Z_{it-1}\}$. We find $\epsilon = 1.05$, which is not statistically different from one.[34] In all cases, we estimate that $\sigma > \epsilon$ with statistical significance (recall our baseline estimate of $\sigma = 3.1$), which from Proposition 1 implies that increases in supplier efficiency S_{it} (or reductions in material costs Z_{it}) have *positive* effects on wages.

Given an estimate of ϵ, labor productivities are then easily recovered as residuals in the relationship between relative input expenditures and prices. Furthermore, the weight on labor in the production function λ is not separately identified from the average level of labor productivity ω_{it} across firms, since both λ and ω_{it} control the productivity of labor relative

[32]The firms used in Table 3 is a subset of the firm baseline sample. The sample is more restrictive because it conditions on firms surviving over 5 years and it conditions on firms for which we can measure both shock. The firms in the sample from Table 3 are larger and less labor intensive than the firms from the firm baseline sample.

[33]We explore heterogeneity in the elasticity of substitution across sectors in Appendix E.2. We aggregate all sectors into 4 groups (primary, manufacturing, trade, services) and find that all elasticities are statistically greater than 1. Furthermore, the elasticity is largest in the manufacturing sector and lowest in services.

[34]Oberfield and Raval (2019) and Doraszelski and Jaumandreu (2018) estimate values of ϵ below one using US and Spanish data respectively. However, their measures of factor prices differ fundamentally from ours. Both papers use average wages in place of W_{it}, while Oberfield and Raval (2019) use an industry fixed effect in place of Z_{it} and Doraszelski and Jaumandreu (2018) use a weighted-average of intermediate input prices in place of Z_{it}. Thus, our estimates, which are based on constructed price indices, are not strictly comparable. Nevertheless, in Column 3, we move closer to the empirical specification in Doraszelski and Jaumandreu (2018) by using the average wage instead of our model-based labor price index. Our estimate of ϵ falls and becomes more similar to their estimates.

to materials. Hence, we set λ to an arbitrary constant in the interval $(0,1)$.

Table 3: Estimation of labor-materials substitution elasticity, ϵ

	$\log E^M/E^L$		
	(1)	(2)	(3)
$\log W/Z$	0.553	0.623	
	(0.058)	(0.094)	
$\log \bar{w}/Z$			0.052
			(0.043)
ϵ	1.55	1.62	1.05
Model for Wage Component	BLM	AKM	Average
Instruments	$\{E^M_{it-1}, E^L_{it-1}\}$	$\{E^M_{it-1}, E^L_{it-1}, W_{it-1}, Z_{it-1}\}$	$\{W_{it-1}, Z_{it-1}\}$
Instrument Polynomial	Quadratic	Linear	Quadratic
First Stage F-Stat	130	84	186
Hansen's J Test	0.121	0.379	0.003
Number of Observations	44,967	44,967	44,967

Notes: This table presents estimates of equation (6.10) using the baseline firm-level dataset. Column 1, our preferred specification, is based on the specification selection criteria described in section 6.2.5. Column 2 uses the AKM wage model to estimate the firm effect W_{it} while Column 3 uses the average firm wage instead of W_{it}. All specifications are estimated using two-stage GMM with a robust weighting matrix. We use the firm baseline sample described in Section 2. Standard errors are shown in parentheses.

6.2.6 Amenities

We identify firm amenities from variation in employment shares that is unexplained by differences in observed wages. Just as we restrict production complementarities θ_i to vary only by a firm's earnings cluster $k(i)$, we impose a similar restriction on amenities to reduce the dimension of parameters that need to be estimated:

$$g_i(\bar{a}) = \tilde{g}_i \bar{g}_{k(i)}(\bar{a}) \tag{6.11}$$

where $\bar{g}_{k(i)}(\bar{a})$ allows for worker-firm variation in amenities but restricts this to be the same for firms within a cluster, while $\tilde{g}_i$ allows for variation in amenities across firms within a cluster. These two components of amenities can then then be identified from:

$$\bar{g}_k(\bar{a}) = \frac{1}{(\bar{a}_m)^{\theta_k}} \left[\Lambda_{kt}(\bar{a})\right]^{\frac{1}{\gamma}}, \qquad \tilde{g}_i = \frac{1}{W_{it}} \left(\bar{\Lambda}_{it}\right)^{\frac{1}{\gamma}} \tag{6.12}$$

where $\Lambda_{kt}(\bar{a})$ is the share of workers of permanent ability $\bar{a}$ employed by firms in earnings cluster k and $\bar{\Lambda}_{it}$ is the share of employment of all worker types by firms in cluster $k(i)$ accounted for by firm i.[35] Since a firm with a high value of amenities is able to attract a large share of workers at a lower wage, amenities are intuitively identified from employment shares after controlling for relevant determinants of earnings – $\bar{a}^{\theta_k}$ at the cluster-ability level and W_{it} at the firm-level.

For a given worker type, we find lower amenity values at larger firms, with this negative relationship being stronger for workers of higher permanent ability. Furthermore, our estimates of amenities and production complementarities jointly imply the sorting of high-ability workers to firms with high earnings premia.[36]

6.2.7 Firm TFP

We identify firm TFPs T_{it} from the firm earnings premia W_{it}. Note that the latter are determined by the equilibrium conditions of the model given firm TFPs and all other identified primitives of the model discussed above. Hence, this give us a set of moments for exact identification of firm TFPs.[37] We choose this approach because it ensures that the model replicates the firm effects on earnings that we estimate from the data, which in turn guarantees that the model matches observed earnings for a given worker conditional on also replicating the worker's observed choice of employer. This allows us to examine changes in labor market outcomes under various counterfactual scenarios with the confidence that the baseline model provides a good fit to observed data. Note that in the limit of our model without intermediates ($\lambda \to 1$), W_{it} is log-linear in T_{it} and hence identification is trivial. With intermediates, however, choosing TFPs to match firm earnings premia requires a numerical solution procedure (see Online Appendix E.6 for details).

[35]See Online Appendix E.3 for a formal derivation.

[36]Further of these findings are relegated to Online Appendix E.3 for brevity.

[37]Formally, these moment conditions can be expressed as $W_{it} = F_i\left(T_t|\Theta_t^{-T}\right)$, where Θ_t is the set of model primitives listed in Definition 1, T_t is the TFP vector, $\Theta_t^{-T} \equiv \Theta_t \setminus T_t$ is the set of identified model primitives other than TFP, and $\{F_i\}_{i\in\Omega^F}$ is a set of known functions that depend on the structural relationships of the model.

7 The Production Network and Inequality Outcomes

We now use the estimated model to investigate the importance of the production network for explaining the observed heterogeneity in firm earnings premia and labor shares highlighted at the start of the paper. In particular, we quantify how production network heterogeneity shapes the following outcomes: (i) the variance of firm earnings premia and worker earnings; (ii) the covariance between firm earnings premia and firm size; (iii) the variance of labor shares of value-added; and (iv) the covariance between labor shares of value-added and firm size. We henceforth refer to these variances and covariances as *inequality outcomes*.

7.1 Numerical solution approach

We begin by solving for a baseline equilibrium in which all model primitives are set to their estimated values (with time-varying primitives averaged across 2005-2010). Note that in our model, key outcomes such as firm earnings premia do not admit closed-form solutions in terms of model primitives. Hence, we require a numerical procedure to solve for model equilibria. Since this is not computationally feasible at the level of individual workers and firms (we have over 6 million workers and over 48 thousand firms), we proceed as follows.

First, we discretize the permanent and transient worker ability distributions into 50 quantiles each, which gives us 2,500 worker types. We then set primitives for each worker type (i.e., abilities and amenities) equal to the corresponding average across workers of each type. Second, within each of the ten firm earnings clusters (see section 6.2.2), we again cluster firms into ten subclusters via a K-means clustering algorithm targeting primitives $\{\omega_{it}, \psi_{it}, \tilde{g}_i\}$ that have been estimated at the firm-level. This gives us 100 firm cluster-subcluster pairs that we henceforth simply refer to as firm *groups*. We then set primitives for each firm group equal to the corresponding average across firms within each group, except for TFP, which we solve for numerically at the firm group level. Finally, for the production network, we measure the fraction of potential buyer-seller firm pairs that are active between each group of buyers b and each group of sellers s in the average year, denoting this by m_{bs}. We then assume that each buyer in b matches with a random fraction m_{bs} of suppliers in s. We also set relationship productivity residuals for each buyer-seller group pair to the corresponding average across active relationships between each pair.

Using this approach, we solve for the model's equilibrium at the worker type and firm group level using a numerical solution algorithm.[38] The correlations between the data and model in terms of firm earnings effects, average wages, employment, sales, labor share of value-added, labor share of cost, downstream access, and upstream access are all between 0.87 and 1.0, indicating that the model provides a good fit to all the key moments even after discretizing worker and firm primitives.[39] By implication, the model closely replicates all of the inequality outcomes observed in the data.

7.2 Sources of variation and counterfactual approach

All inequality outcomes in the model are driven by heterogeneity in the following worker and firm primitives: (i) the production network, Ω_{it}^{S}; (ii) relationship productivity residuals, $\tilde{\psi}_{ijt}$ (iii) firm productivities, $\{T_{it}, \omega_{it}, \psi_{it}\}$; (iv) production complementarities, θ_i; (v) amenities, $g_i(\cdot)$; (vi) and worker abilities, $\{\bar{a}_m, \hat{a}_{mt}\}$. We refer to each of these six sets of primitives as a *source of variation* in the model.

To quantify the contribution of each source of variation to a given inequality outcome, we proceed as follows. First, we simulate counterfactual equilibria of the model in which each source of variation v is eliminated by setting its value for all workers or firms equal to the mean of v across the respective sample. We then compute inequality outcomes and compare these to the baseline equilibrium, taking the difference between these values as a measure of the contribution of the source of variation v to each inequality outcome. For the production network, we do this separately for heterogeneity across suppliers and customers. For example,

[38] This is described formally in Online Appendix F.

[39] See Figure 5 in the Online Appendix for details. We observe a greater discrepancy between the model and data for labor shares of value-added, largely due to the fact that we do not consider firm heterogeneity in output markups. Even in this case, however, the model fit is good (correlation coefficient of 0.87).

to eliminate heterogeneity across suppliers, we replace the observed network at the buyer group (b) and seller group (s) level, m_{bs}, with a counterfactual network $\hat{m}^{S}_{bs} = \frac{\sum_{s'} m_{bs} N_{s'}}{\sum_{s'} N_{s'}}$ that is randomized across suppliers while holding constant the total supplier count for each firm group, where $N_{s'}$ denotes the number of firms in seller group s'. We follow an analogous procedure to eliminate heterogeneity in customer matching.

Note that eliminating a source of variation v not only removes variation in outcomes arising from v but also from the covariance between v and all other sources of variation. Therefore, any changes in inequality outcomes that arise from eliminating v cannot be attributed to v alone. To address this, we adopt a Shapley-based approach: we simulate counterfactuals by eliminating *all* possible combinations of the sources of variation listed above and then compute the Shapley value for each source of variation in terms of its effect on inequality outcomes.[40] Intuitively, this provides an *average* measure of the change in each inequality outcome when a source of variation is eliminated, under all possible combinations of the remaining sources of variation. This approach has two advantages. First, it accounts for interdependencies between sources of variation. Second, it ensures that each inequality outcome is decomposed *exactly* into contributions from each source of variation.

7.3 Results

Table 4 presents our findings. Each panel shows results for a different inequality outcome, with the values in each panel reporting the shares of the inequality outcome accounted for by the indicated sources of variation.

First, consider the role of production network heterogeneity in explaining differences in earnings. In panel (a), around one-third (30.2%) of the variation in log earnings premia across firms (weighted by employment) is accounted for by network heterogeneity, with supplier

[40] To illustrate, consider two sources of variation, Θ_A and Θ_B, and suppose that an inequality outcome such as the variance of earnings can be expressed as $\text{var}(\Theta_A) + \text{var}(\Theta_B) + 2\text{cov}(\Theta_A, \Theta_B)$. Eliminating Θ_A reduces the variance of earnings by $\delta_{A1} = \text{var}(\Theta_A) + 2\text{cov}(\Theta_A, \Theta_B)$. Eliminating Θ_A when Θ_B has already been eliminated reduces the variance of earnings by $\delta_{A2} = \text{var}(\Theta_A)$. The Shapley contribution of Θ_A to earnings variance is then $\frac{\delta_{A1}+\delta_{A2}}{2} = \text{var}(\Theta_A) + \text{cov}(\Theta_A, \Theta_B)$. See Appendix E for a formal definition of the Shapley value.

Table 4: Decomposition of inequality outcomes

(a) variance, log firm earnings premia (baseline = 0.18)					
supplier network:	23.6%	customer network:	6.6%	firm productivities:	40.7%
prod. complementarities:	26.7%	firm amenities:	13.3%	worker abilities:	-10.8%
(b) variance, log worker earnings (baseline = 0.64)					
supplier network:	9.7%	customer network:	3.2%	firm productivities:	16.6%
prod. complementarities:	1.7%	firm amenities:	1.1%	worker abilities:	67.8%
(c) covariance, log firm earnings premia and log sales (baseline = 0.57)					
supplier network:	35.3%	customer network:	15.8%	firm productivities:	44.8%
prod. complementarities:	8.0%	firm amenities:	1.7%	worker abilities:	-5.5%
(d) variance, labor share of value-added (baseline = 0.02)					
supplier network:	21.9%	customer network:	4.2%	firm productivities:	76.2%
prod. complementarities:	-1.7%	firm amenities:	-1.1%	worker abilities:	0.5%
(e) covariance, labor share of value-added and log sales (baseline = -0.09)					
supplier network:	-3.8%	customer network:	71.9%	firm productivities:	30.9%
prod. complementarities:	4.5%	firm amenities:	0.2%	worker abilities:	-3.8%

Notes: Each panel shows results for a different inequality outcome, with the value of the outcome in the baseline equilibrium reported in the panel headers. The variance in (a) and covariance in (c) are weighted by firm employment, while the variance in (d) and covariance in (e) are weighted by firm value-added. In each panel, the values reported are the shares of each inequality outcome accounted for by the corresponding source of variation.

heterogeneity (23.6%) playing a more important role than customer heterogeneity (6.6%). Note that randomizing the identity of firms' suppliers while holding constant the number of suppliers for each firm also eliminates heterogeneity in the number of *customers* per firm. Hence, our finding that supplier heterogeneity matters more than customer heterogeneity for variation in earnings premia is consistent with the fact documented in Table A.II that the dispersion in the number of buyers is larger than the dispersion in the number of suppliers. These results are also consistent with findings by Bernard et al. (2022), who show that variation in firm sales is largely explained by differences in the number of customers per firm. Overall, we find that network heterogeneity is a key driver of differences in employer-specific earnings premia.

In panel (b), we see that production network heterogeneity also explains 12.9% of the variance of log earnings across workers, with supplier heterogeneity (9.7%) again playing a larger role than customer heterogeneity (3.2%). In comparison, own-firm primitives (productivities, production complementarities, and amenities) explain 19.3% of log earnings variance, while worker abilities explain the remainder (67.8%). Hence, network heterogeneity accounts for around two-fifths ($\frac{12.9}{12.9+19.3}$) of the variance of log worker earnings that is unexplained by worker characteristics.[41]

Second, consider the role of the production network in explaining the positive firm size wage premium. In panel (c), 51.1% of the positive covariance between log earnings premia and log sales across firms (weighted by employment) is explained by network heterogeneity, 35.3% from supplier heterogeneity and 15.8% from customer heterogeneity. In other words, about half of the firm size wage premium is attributable to differences in production network linkages. Intuitively, this occurs because better access to suppliers and customers in the production network leads to increases in both firm size and firm earnings effects. Hence, network heterogeneity amplifies the positive relationship between these two outcomes.

Third, consider the role of the production network in explaining differences in labor shares across firms. In panel (d), 26.1% of the variation in labor shares of value-added across firms (weighted by value-added) is explained by network heterogeneity, with most of this accruing from supplier heterogeneity (21.9%) rather than customer heterogeneity (4.2%). As discussed in section 5.3, network heterogeneity affects the labor share of value-added through the relative cost of labor to materials, W_{it}/Z_{it}. The identity of a firm's suppliers matters most for its upstream access, which has a direct impact on Z_{it} while also having an indirect effect on W_{it}. On the other hand, the identity of a firm's customers only affects labor shares through an indirect effect on W_{it}.

[41]Note that network heterogeneity shapes earnings inequality not only through the variance in firm earnings effects, but also through the covariance between firm effects and worker effects. We find that network heterogeneity accounts for 20.4% of this covariance, which in turn explains 19.8% of the variance in log worker earnings (see Online Appendix A for details).

Finally, consider the role of the production network in explaining why larger firms tend to have lower labor shares of value-added. In panel (e), we see that 68.1% of the negative covariance between labor shares of value-added and log sales across firms (weighted by value-added) is explained by network heterogeneity. As shown in Fact 2 of the motivation, larger firms tend to have lower labor shares of value-added and better upstream and downstream network access in the baseline equilibrium. Furthermore, as discussed above, improvements in upstream and downstream access both tend to lower labor shares. Hence, the advantage that large firms tend to have in terms of network access amplifies the negative relationship between firm size and labor shares of value-added.

In sum, we find that production network heterogeneity plays a key role in explaining all four inequality outcomes. To better understand what drives these findings, recall that all variation across firms can be fully described in terms of two sets of characteristics: exogenous firm-specific primitives $\{T_{it}, \omega_{it}, \phi_i, g_i\}$ and endogenous network access measures $\{D_{it}, S_{it}\}$. Now, consider two workers with identical abilities a who are employed at two firms that are identical in terms of their exogenous firm-specific primitives. These two workers can still earn different wages if their employers differ in terms of their network access measures, because these characteristics affect a firm's MRPL and hence its wage premium through the scale and substitution effects discussed above following Proposition 1. Heterogeneity in the network access measures is in turn driven by heterogeneity in the network connections that a firm has with its customers and suppliers, Ω_{it}^C and Ω_{it}^S, and the productivities of these relationships, $\{\psi_{jit}\}_{j\in\Omega_{it}^C}$ and $\{\psi_{ijt}\}_{j\in\Omega_{it}^S}$.

The key model parameters that affect how production network heterogeneity translates into differences in earnings are the same as those that drive the scale and substitution effects in Proposition 1. The positive scale effect of greater downstream and upstream access on earnings is captured by the scale elasticity $\Gamma_{it} \equiv \frac{1}{\gamma+\sigma\left(1-s_{it}^M\right)+\epsilon s_{it}^M}$, which is greater when: (i) labor supply is less elastic (smaller γ), so that firms have to increase wages more in order to grow their workforce; (ii) firms have more output market power (smaller σ), so that they can grow more in response to greater network access; and (iii) labor and materials are less substitutable (smaller ϵ), so that firms find it more difficult to escape from increasing marginal costs of labor by purchasing materials at constant marginal cost as they grow. In addition, upstream access has a greater scale effect on earnings when materials are more important in production (larger s_{it}^M, which is in turn driven by a smaller value of λ and larger values of ω_{it}). The negative substitution effect of greater upstream access on earnings, on the other hand, is stronger when labor and material are more substitutable (larger ϵ).

Note also from Table A.II that there is more heterogeneity in the number of buyers across firms than in the number of suppliers. This helps to explain our finding that heterogeneity in the supplier network tends to matter for differences in earnings and labor shares compared with heterogeneity in the customer network, because randomizing the supplier network eliminates differences in the number of buyers across firms, as explained above.

7.4 Sensitivity of results to value-added production functions

We have highlighted above the importance of relaxing the assumption of value-added production functions for understanding how workers gain from firm growth. How important is relaxing this assumption for the extent to which production networks contribute to inequality outcomes? To assess this, we first set the labor-materials substitution elasticity to $\epsilon = 1$ instead of our preferred estimate of $\epsilon = 1.5$, since this is necessary for a value-added representation of the production function to be valid. Second, since labor productivity ω_{it} is not separately identified from TFP when $\epsilon = 1$, we set $\omega_{it} = 1$ for all firms. Third, since labor shares of cost are determined by λ (the weight on labor in the production function) when $\epsilon = 1$, we choose this parameter to match the aggregate labor share of cost. Fourth, since our TFP estimates depend on the value of ϵ, we re-estimate TFP. Finally, we recompute the decompositions of inequality outcomes shown in Table 4.

It is immediately obvious that the model with $\epsilon = 1$ cannot speak to labor share heterogeneity, since Cobb-Douglas technology imposes common labor shares of cost and therefore common labor shares of value-added across firms. Hence, we consider here only the variance of log firm earnings premia, the variance of log worker earnings, and the covariance between log firm earnings premia and log sales (panels (a)-(c) of Table 4). We find that the share of each of these inequality outcomes accounted for by production network heterogeneity is substantially different under $\epsilon = 1$ compared with our baseline. Specifically, under Cobb-Douglas technology, production network heterogeneity explains 51.2% of the variance in log firm earnings premia (baseline: 30.2%), 21.1% of the variance in log worker earnings (baseline: 12.9%), and 75.2% of the covariance between log firm earnings premia and log sales (baseline: 51.1%). Hence, by these metrics, our estimated production function with $\epsilon = 1.5$ is far from being well-approximated by a value-added production function.

8 Conclusion

We have developed in this paper a unifying framework with firm heterogeneity in both earnings premia and labor shares of value-added. Central to our framework are firm labor market power and heterogeneous firm-to-firm production network linkages with CES production technologies. These features allow the model to reconcile why firms with greater downstream and upstream access have higher earnings premia and lower labor shares of value-added and cost, as well as why workers benefit incompletely and differentially from shocks to customer demand and supplier efficiency. Using linked employer-employee and firm-to-firm transactions data from Chile, we structurally estimate the model and show in particular how these data can be used to identify the elasticity of substitution between labor and materials when these inputs are heterogeneous within firms. Counterfactual simulations of our estimated model indicate that production network heterogeneity is an important driver of key inequality outcomes, in particular the variances of worker earnings, firm earnings premia, and labor shares of value-added, as well the covariances between firm earnings premia, labor shares of value-added, and firm size.

We conclude with three potential directions for future research on the interaction between workers and production networks.

First, there are potentially other mechanisms that could make the production network matter for earnings inequality. For example, there could be complementarities between buyer and supplier characteristics that matter for wage premia, as in Demir et al. (2024), and shocks could lead to changes in the the matching patterns between these firms. Hypothetically, greater positive assortative matching in the presence of buyer-seller complementarities could amplify differences across firms and hence lead to greater earnings inequality. To examine this in the context of our paper, one would need to develop a tractable way to model endogenous network formation and buyer-seller complementarities in the presence of monopsony power in the labor market.

Second, there is growing evidence that worker outsourcing is a key driver of increases in earnings inequality (Goldschmidt and Schmieder (2017)). However, in these settings, it is typically not possible to directly observe and hence measure outsourcing. The growing availability of linked employer-employee and firm-to-firm datasets provides a unique opportunity to measure flows of both goods and workers between firms. This will allow researchers to more accurately measure outsourcing at the firm and to understand its incidence on both workers and firms.

Third, there is growing interest among both policymakers and researchers in understanding the effects of automation on worker outcomes. It is natural to view these effects as arising from the substitution of labor by inputs such as industrial robots. For example, Acemoglu and Restrepo (2020) estimate the effects of increased robot usage on employment and wages in US labor markets, finding robust negative effects. More recent theoretical work by Jackson and Kanik (2020) develops a model of robot-labor substitution that accounts for production network linkages between firms. A quantitative study of the mechanisms highlighted by this literature using matched employer-employee and firm-to-firm transactions data is therefore likely to yield important insights.

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A Data Details

A.1 Data cleaning

To clean the firm-to-firm trade dataset, we drop relationships involving firms that do not report value-added or employment, or firms that report negative value-added, sales, or materials. We also follow Bernard et al. (2022) and iteratively drop firms that have only one relationship, which is required for a decomposition of firm-to-firm transaction values into buyer and seller effects that we describe below.

To clean the employer-employee dataset, we impose sample restrictions following the criteria outlined in Lamadon et al. (2022). In each year, we start with all individuals aged 25-60 who are linked to at least one employer. We identify links using only information on labor contracts (tax affidavit 1887). Next, we drop firms that have missing or negative value-added, sales, or materials in the balance sheet data (tax form 29). Then, we keep for each worker the firm that pays the highest earnings in a given year. Since we do not have hours worked or a direct measure of full-time employment, we follow the literature by including workers for whom annual earnings are above a minimum threshold (Song et al., 2019). We set the threshold equal to 32.5% of the national average of earnings in order to make our estimates comparable to the cross-country study of earnings inequality in Bonhomme et al. (2020).

Finally, in terms of confidentiality, we merge these data sets using unique tax IDs of workers and firms that are common across sources. To secure the privacy of workers and firms, the Chilean IRS requires all results that are published to be calculated using at least 25 unique tax IDs. All the analysis was implemented by the authors and did not involve nor compromise the Chilean IRS. Officials of the Central Bank of Chile processed the disaggregated data from the Chilean IRS. The information contained in the databases of the Chilean IRS is of a tax nature originating in self-declarations of taxpayers presented to the Service; therefore, the veracity of the data is not the responsibility of the Service.

A.2 Sample sizes and descriptive statistics

Table A.I provides sample size information for our baseline firm-to-firm dataset, employer-employee dataset (including the movers and stayers subsamples), and firm-level dataset, which are defined in section 2. Table A.II provides basic descriptive statistics about these datasets.

Table A.I: Overview of Sample Sizes

	(1)	(2)	(3)	(4)	(5)
Dataset	**Employer-Employee**			**Firm-to-Firm**	**Firm**
	Baseline	**Movers**	**Stayers**	**Baseline**	**Baseline**
Number of Workers	6,496,849	6,183,692	724,957		1,969,845
Number of Worker-Year Observations	41,954,008	40,130,960	6,571,483		3,559,903
Number of Firms	487,504	200,592	5,726	151,817	44,967
Number of Firm-Year Observations	2,315,927	1,378,554	61,823	485,566	125,726
Number of Firm-to-Firm Links				16,831,546	4,578,626
Number of Firm-to-Firm-Year Observations				31,743,495	6,233,839

Notes: This table provides an overview of the samples used throughout the paper.

Table A.II: Descriptive Statistics of Datasets

	(1)	(2)	(3)	(4)	(5)
Dataset	**Employer-Employee**			**Firm-to-Firm**	**Firm**
Panel A: Worker Characteristics	**Baseline**	**Movers**	**Stayers**	**Baseline**	**Baseline**
Mean Log Worker Earnings (Log US $)	9.36	9.38	9.74		9.17
Median Log Worker Earnings (Log US $)	9.25	9.27	9.66		9.02
Mean Worker Age	40.2	40.1	42.6		39.3
Median Worker Age	39.4	39.4	42.6		38.5
Panel B: Firm Characteristics	**Baseline**	**Movers**	**Stayers**	**Baseline**	**Baseline**
Mean Number of Workers	9	20	281		27
Median Number of Workers	2	4	94		7
Mean Wage Bill per Worker (US $)	10,199	11,145	7,833		9,440
Median Wage Bill per Worker (US $)	6,943	8,323	6,672		7,103
Mean Value Added per Worker (US $)	56,315	58,610	50,077		49,604
Median Value Added per Worker (US $)	23,424	25,659	26,583		23,389
Mean Log Value Added (Log US $)	11.0	11.8	14.6		12.2
Median Log Value Added (Log US $)	11.0	11.7	14.8		12.1
Mean Labor Share	0.49	0.45	0.70		0.42
Median Labor Share	0.32	0.34	0.21		0.34
Panel C: Production Network Characteristics	**Baseline**	**Movers**	**Stayers**	**Baseline**	**Baseline**
Mean Number of Suppliers				35	67
Median Number of Suppliers				19	36
Standard Dev. of Log Number of Suppliers				1.01	0.9
Mean Number of Buyers				34	80
Median Number of Buyers				4	8
Standard Dev. of Log Number of Buyers				1.5	1.4
Mean Materials Share of Sales				0.57	0.58
Median Materials Share of Sales				0.61	0.61
Mean Intermediate Share of Sales				0.38	0.40
Median Intermediate Share of Sales				0.33	0.38
Number of Firms	487,504	200,592	5,726	151,817	44,967

Notes: This table provides descriptive statistics of all the samples used in the paper.

B Proofs of Propositions

Throughout this section, we omit time subscripts for brevity.

B.1 Proof of Proposition 1

As discussed in section 4.5, a firm's earnings premium W_i is completely determined by its demand shifter D_i, supplier efficiency S_i, TFP T_i, labor productivity ω_i, and sorting composite $\bar{\phi}_i$. We begin by deriving comparative statics for W_{it} with respect to these variables. Recall that $\hat{Y}_i$ denotes the marginal log change in some variable Y_i.

Totally differentiating (4.13), (4.15)-(4.16) and solving for $\left\{\hat{W}_i, \hat{X}_i, \hat{\nu}_i\right\}$, we obtain:

$$\hat{W}_i = \Gamma_i \hat{D}_i + (\sigma - \epsilon)\, \varepsilon_i^M \Gamma_i \hat{S}_i + (\sigma - 1)\, \Gamma_i \hat{T}_i + \left[\sigma - 1 - (\sigma - \epsilon)\, \varepsilon_i^M\right] \Gamma_i \hat{\omega}_i - \Gamma_i \hat{\bar{\phi}}_i \tag{B.1}$$

$$\hat{X}_i = \left(\gamma + \epsilon \varepsilon_i^M\right) \Gamma_i \hat{D}_i + \sigma (\gamma + \epsilon)\, \varepsilon_i^M \Gamma_i \hat{S}_i + \sigma \left(\gamma + \epsilon \varepsilon_i^M + 1 - \varepsilon_i^M\right) \Gamma_i \hat{T}_i \tag{B.2}$$

$$+ \sigma \left(1 - \varepsilon_i^M\right) (1 + \gamma)\, \Gamma_i \hat{\omega}_i + \sigma \left(1 - \varepsilon_i^M\right) \Gamma_i \hat{\bar{\phi}}_i$$

$$\hat{\upsilon}_i = \epsilon \Gamma_i \hat{D}_i + \epsilon (\gamma + \sigma)\, \Gamma_i \hat{S}_i + \epsilon (\sigma - 1)\, \Gamma_i \hat{T}_i - \epsilon (1 + \gamma)\, \Gamma_i \hat{\omega}_i - \epsilon \Gamma_i \hat{\bar{\phi}}_i \tag{B.3}$$

where ε_i^M denotes the elasticity of the CES aggregator F with respect to materials evaluated at $(1, \nu_i)$ and $\Gamma_i \equiv \frac{1}{\gamma + \sigma\left(1 - \varepsilon_i^M\right) + \epsilon \varepsilon_i^M}$ is the scale elasticity for firm i. Furthermore, from equation (4.19), we can express the material share of cost (adjusted for markdowns on wage) as $s_i^M = \frac{\nu_i}{W_i S_i / \omega_i + \nu_i}$. In addition, using the first-order conditions (4.13) and (4.15), as well as the fact that $F(1, \nu) = F_M(1, \nu)\, \nu + F_L(1, \nu)$ since F is homogeneous of degree one, we have $\varepsilon_i^M = \frac{\nu_i}{W_i S_i / \omega_i + \nu_i}$. Hence, $\varepsilon_i^M = s_i^M$, so that the elasticity of F with respect to materials is equal to the material share of cost in equilibrium. This allows us to write the scale elasticity as $\Gamma_i \equiv \frac{1}{\gamma + \sigma\left(1 - s_i^M\right) + \epsilon s_i^M}$, which is strictly greater than zero. Note that in equation (B.1), the coefficient on $\hat{D}_i$ is thus strictly positive, while the coefficient on $\hat{S}_i$ is strictly positive if and only if $\sigma > \epsilon$.

Next, we characterize the pass-through of shocks in the production network into firm earnings effects more generally, including the pass-through of shocks across firms. Now, let $\hat{Y}$ denote the vector of marginal log changes $\left\{\hat{Y}_i\right\}_{i \in \Omega^F}$ across firms for some variable Y. Totally differentiating the expressions in equation (4.10) gives:

$$\hat{D} = S^{sales} \hat{\Delta}, \qquad \hat{S} = \frac{1}{\sigma - 1} S^{mat} \hat{\Phi} \tag{B.4}$$

where we have used the fact that the share of firm i's sales accounted for by firm j can be expressed as $s_{jit}^{sales} \equiv \frac{R_{jit}}{\sum_{k \in \Omega_{it}^C \cup \{F\}} R_{kit}} = \frac{\Delta_j}{D_i}$, while the share of firm i's input expenditures accounted for by firm j can be expressed as $s_{ijt}^{mat} \equiv \frac{R_{ijt}}{\sum_{k \in \Omega_{it}^S} R_{ikt}} = \frac{\Phi_{jt} \psi_{ijt}}{Z_{it}^{1-\sigma}}$. Then, totally differentiating (4.11) and using (4.19), we can express marginal changes in buyer and seller effects as:

$$\hat{\Delta} = \gamma \hat{W} - \sigma \hat{S} + \hat{\nu} + \hat{\omega}, \qquad \hat{\Phi} = \frac{\sigma - 1}{\sigma}\left(\hat{X} - \hat{D}\right) \tag{B.5}$$

Furthermore, taking the ratio of the first-order conditions for the profit-maximization problem (4.13) and (4.15) and totally differentiating gives:

$$\hat{W} + \hat{S} = \epsilon^{-1}\hat{\nu} + \hat{\omega} \tag{B.6}$$

Combining (B.4), (B.5), and (B.6), we obtain the following expressions for marginal changes in demand shifters and supplier efficiencies:

$$\hat{D} = S^{sales}\left[\left(\gamma + \epsilon\right)\hat{W} - \left(\sigma - \epsilon\right)\hat{S} + \left(1 - \epsilon\right)\hat{\omega}\right], \qquad \hat{S} = \frac{1}{\sigma} S^{mat}\left(\hat{X} - \hat{D}\right) \tag{B.7}$$

Now equations (B.1)-(B.3), and (B.7) define a linear system in $\left\{\hat{W}, \hat{X}, \hat{\nu}, \hat{D}, \hat{S}\right\}$, given changes in TFP $\hat{T}$ and labor productivity $\hat{\omega}$. Eliminating $\hat{X}$ and $\hat{\nu}$ from this system, we can write the remaining equations as:

$$\hat{W} = H^{WT}\hat{T} + H^{W\omega}\hat{\omega} + H^{WD}\hat{D} + H^{WS}\hat{S} \tag{B.8}$$

$$\hat{D} = S^{sales}\left[H^{DT}\hat{T} + H^{D\omega}\hat{\omega} + H^{DD}\hat{D} + H^{DS}\hat{S}\right] \tag{B.9}$$

$$\hat{S} = S^{mat}\left[H^{ST}\hat{T} + H^{S\omega}\hat{\omega} + H^{SS}\hat{S} + H^{SD}\hat{D}\right] \tag{B.10}$$

where the H terms are all diagonal matrices. The matrices capturing the dependence of

$\left\{\hat{W},\hat{D},\hat{S}\right\}$ on productivity shocks $\left\{\hat{T},\hat{\omega}\right\}$ have i^{th}-diagonal elements given by:

$$H_i^{WT} = (\sigma-1)\,\Gamma_i, \qquad H_i^{W\omega} = \left[(\sigma-1)-(\sigma-\epsilon)\,s_i^M\right]\Gamma_i \tag{B.11}$$

$$H_i^{DT} = (\gamma+\epsilon)\,(\sigma-1)\,\Gamma_i, \qquad H_i^{D\omega} = (1+\gamma)\,(\sigma-\epsilon)\left(1-s_i^M\right)\Gamma_i \tag{B.12}$$

$$H_i^{ST} = \left(\gamma+1-s_i^M+\epsilon s_i^M\right)\Gamma_i, \qquad H_i^{S\omega} = (1+\gamma)\left(1-s_i^M\right)\Gamma_i \tag{B.13}$$

while the matrices capturing the interrelation between $\left\{\hat{W},\hat{D},\hat{S}\right\}$ have i^{th}-diagonal elements given by:

$$H_i^{WD} = \Gamma_i, \qquad H_i^{WS} = (\sigma-\epsilon)\,s_i^M\Gamma_i \tag{B.14}$$

$$H_i^{DD} = (\gamma+\epsilon)\,\Gamma_i, \qquad H_i^{DS} = -\,(\sigma-\epsilon)\,(\gamma+\sigma)\left(1-s_i^M\right)\Gamma_i \tag{B.15}$$

$$H_i^{SD} = -\left(1-s_i^M\right)\Gamma_i, \qquad H_i^{SS} = (\gamma+\epsilon)\,s_i^M\Gamma_i \tag{B.16}$$

Now, first note the existence of *feedback effects* arising from the fact that marginal costs are increasing with scale due to the upward-sloping labor supply curves faced by each firm. These feedback effects go in two directions. To illustrate, consider a simple supply chain $j_s \to i \to j_c$, where arrows indicate the flow of goods. First, consider a positive demand shock to customer j_c. This leads to an increase in demand D_i for firm i (H^{DD}), which not only affects firm i's earnings premium (H^{WD}), but also leads to an increase in marginal cost and hence in the output price for firm i, thus lowering the supplier efficiency for customer j_c (H^{SD}). This in turn has a feedback effect on the demand from customer j_c (H^{DS}). Second, consider an increase in supplier efficiency for supplier j_s. This raises the supplier efficiency for firm i (H^{SS}), which not only affects firm i's earnings premium (H^{WS}), but also affects the demand for materials by firm i and hence the demand faced by supplier j_s (H^{DS}). This in turn has a feedback effect on the marginal cost and output price of the supplier, and hence on the supplier efficiency for firm i (H^{SD}).

In sum, feedback effects stemming from scale-dependent marginal costs are captured by elements of the product $H^{SD}H^{DS}$ (which is symmetric, given that the H matrices are diagonal). Given our estimates of $\{\gamma, \sigma, \epsilon\}$ and evaluating firm-specific material cost shares at the median value in the average year in our sample, the magnitudes of the elements of these matrices are approximately $H_i^{DS} \approx 0.29$ and $H_i^{SD} \approx 0.02$, so that the feedback elasticity is approximately 0.6%. Hence, feedback effects are likely to be small empirically. Ignoring these feedback effects by setting H^{DS} and H^{SD} to zero in equations (B.9) and (B.10) and solving for $\hat{W}$ as a function of $\hat{D}$ and $\hat{S}$ then gives the pass-through expressions in Proposition 1. More generally, it is straightforward to solve the linear system (B.8)-(B.10) for $\hat{W}$ including feedback effects, with coefficients that can be fully determined given estimates of $\{\gamma, \sigma, \epsilon\}$, network shares $\left\{S^{sales}, S^{mat}\right\}$, and material shares of cost s_i^M.

B.2 Proof of Proposition 2

Note from equations (5.6) and (5.7) that a firm's labor shares of cost and value-added are strictly decreasing in W_iS_i if $\epsilon > 1$, strictly increasing in W_iS_i if $\epsilon < 1$, and independent of W_iS_i if $\epsilon = 1$. Hence, we need only to show that (i) W_iS_i is strictly increasing in D_i and (ii) W_iS_i is strictly increasing in S_i. From equation (B.1), we have $\frac{\partial \log W_i}{\partial \log D_i} = \Gamma_i > 0$, which immediately establishes (i). To establish (ii), note from equation (B.1) that $\frac{\partial \log W_i}{\partial \log S_i} = (\sigma - \epsilon)\, s_i^M \Gamma_i$. Hence:

$$\frac{\partial \log (W_iS_i)}{\partial \log S_i} = (\sigma - \epsilon)\, s_i^M \Gamma_i + 1 \tag{B.17}$$

$$= (\gamma + \sigma)\, \Gamma_i \tag{B.18}$$

$$> 0 \tag{B.19}$$

B.3 Proof of Proposition 3

Using equations (4.13), (4.15), (4.16), (4.17), and (4.19) we can write value-added and value-added per worker for firm i as:

$$VA_i = \eta^{\gamma} W_i^{1+\gamma} \bar{\phi}_i \left[\mu + \frac{1}{\sigma - 1} \left(\frac{1-\lambda}{\lambda} \right) \left(\frac{W_i S_i}{\omega_i} \right)^{\epsilon - 1} \right] \tag{B.20}$$

$$VAPW_i = W_i \left(\frac{\bar{\phi}_i}{\tilde{\phi}_i} \right) \left[\mu + \frac{1}{\sigma - 1} \left(\frac{1-\lambda}{\lambda} \right) \left(\frac{W_i S_i}{\omega_i} \right)^{\epsilon - 1} \right] \tag{B.21}$$

where $\tilde{\phi}_i \equiv \sum_{a \in A} \kappa_i(a) \phi_i(a)^{\gamma}$. Now consider shocks to a firm's demand shifter D_i and supplier efficiency S_i. Log differentiating equation (B.20) with respect to D_i and S_i gives:

$$\frac{\partial \log VA_i}{\partial \log D_i} = (1 + \gamma + \chi_i) \frac{\partial \log W_i}{\partial \log D_i}, \qquad \frac{\partial \log VA_i}{\partial \log S_i} = (1 + \gamma + \chi_i) \frac{\partial \log W_i}{\partial \log S_i} + \chi_i \tag{B.22}$$

where we have defined $\chi_i \equiv \frac{s_i^M (\epsilon - 1)}{s_i^M + \sigma\left(1 - s_i^M\right)}$ for brevity and used the result that the firm's material cost share adjusted for wage markdowns is given by $s_i^M = \left[1 + \frac{\lambda}{1-\lambda} \left(\frac{W_i S_i}{\omega_i} \right)^{1-\epsilon} \right]^{-1}$. Similarly, log differentiating equation (B.21) with respect to D_i and S_i, we obtain:

$$\frac{\partial \log VAPW_i}{\partial \log D_i} = (1 + \chi_i) \frac{\partial \log W_i}{\partial \log D_i}, \qquad \frac{\partial \log VAPW_i}{\partial \log S_i} = (1 + \chi_i) \frac{\partial \log W_i}{\partial \log S_i} + \chi_i \tag{B.23}$$

Then, from equation (4.19), we have:

$$\frac{\partial \log E_i^L}{\partial \log D_i} = (1 + \gamma) \frac{\partial \log W_i}{\partial \log D_i}, \qquad \frac{\partial \log E_i^L}{\partial \log S_i} = (1 + \gamma) \frac{\partial \log W_i}{\partial \log S_i} \tag{B.24}$$

Finally, from equation (B.1), we have:

$$\frac{\partial \log W_i}{\partial \log D_i} = \frac{1}{\gamma + \sigma\left(1 - s_i^M\right) + \epsilon s_i^M}, \qquad \frac{\partial \log W_i}{\partial \log S_i} = \frac{(\sigma - \epsilon) s_i^M}{\gamma + \sigma\left(1 - s_i^M\right) + \epsilon s_i^M} \tag{B.25}$$

Equations (B.22)-(B.25) allow us to solve explicitly for the partial derivatives of VA_i, $VAPW_i$, E_i^L, and W_i with respect to D_i and S_i. The solution for these partial derivatives implies that the relative pass-through coefficients defined in Proposition 3 are given by:

$$\beta_D^{EL/VA} = \frac{1+\gamma}{1+\gamma+\chi_i}, \qquad \beta_S^{EL/VA} = \frac{1+\gamma}{1+\gamma+c_i\chi_i} \tag{B.26}$$

$$\beta_D^{W/VAPW} = \frac{1}{1+\chi_i}, \qquad \beta_S^{W/VAPW} = \frac{1}{1+c_i\chi_i} \tag{B.27}$$

where $c_i \equiv \frac{\gamma+\sigma}{(\sigma-\epsilon)s_i^M} > 0$ since we are assuming $\sigma > \epsilon$. Furthermore, note that $\chi_i > 0$ if $\epsilon > 1$, $\chi_i < 0$ if $\epsilon < 1$, and $\chi_i = 0$ if $\epsilon = 1$. From this, the dependence of the magnitudes of the β pass-through coefficients on ϵ stated in Proposition 3 follows immediately.

Furthermore, taking ratios of the relative pass-through coefficients for demand shocks versus supplier efficiency shocks, we have:

$$\beta_D^{EL/VA}/\beta_S^{EL/VA} = \frac{1+\gamma+c_i\chi_i}{1+\gamma+\chi_i}, \qquad \beta_D^{W/VAPW}/\beta_S^{W/VAPW} = \frac{1+c_i\chi_i}{1+\chi_i} \tag{B.28}$$

Now note that $c_i > 1$ since we are assuming $\sigma > \epsilon$. Hence, the ratios above are strictly increasing in χ_i. Furthermore, these ratios are exactly one when $\chi_i = 0$. From this, the dependence of these ratios on ϵ described in Proposition 3 follows immediately.

C Model extension with fixed costs

Consider the following version of our model with fixed labor and material costs. Suppose that workers are homogeneous for simplicity. A firm's production function is:

$$X(L, M) = T\left[\lambda^{\frac{1}{\varepsilon}}(\phi L)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\lambda)^{\frac{1}{\varepsilon}} M^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{C.1}$$

where L and M are variable labor and material inputs, respectively. Suppose that the firm also has to pay fixed labor and material costs, denoted by L_f and M_f. Hence, the firm's total usage of labor and materials is:

$$\bar{L} = L + L_f \tag{C.2}$$

$$\bar{M} = M + M_f \tag{C.3}$$

The firm faces demand given by:

$$X = Dp^{-\sigma} \tag{C.4}$$

where D is a demand shifter. The firm also faces and an upward-sloping labor supply curve given by:

$$\bar{L} = \kappa W^{\gamma} \tag{C.5}$$

where κ is a supply shifter. The firm's price of materials is Z and we denote by $S \equiv \frac{1}{Z}$ the firm's supplier efficiency.

The firm's profit maximization problem is:

$$\max_{L,M} \left\{ pX - W\bar{L} - \bar{M}/S \right\} \tag{C.6}$$

subject to equations (C.1)-(C.5). The first-order conditions can be written as:

$$\frac{1}{\mu} D^{\frac{1}{\sigma}} \left[X(L,M) \right]^{\frac{\sigma-1}{\sigma}} \left[1 - s_M(L,M) \right] = \frac{1}{\eta} \left(\frac{L + L_f}{\kappa} \right)^{\frac{1}{\gamma}} \tag{C.7}$$

$$\frac{1}{\mu} D^{\frac{1}{\sigma}} \left[X(L,M) \right]^{\frac{\sigma-1}{\sigma}} s_M(L,M) = \frac{1}{S} \tag{C.8}$$

where $\mu \equiv \frac{\sigma}{\sigma-1}$, $\eta \equiv \frac{\gamma}{\gamma+1}$, and $s_M(L,M) \equiv \frac{(1-\lambda)^{\frac{1}{\varepsilon}} M^{\frac{\varepsilon-1}{\varepsilon}}}{\lambda^{\frac{1}{\varepsilon}} (\phi L)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\lambda)^{\frac{1}{\varepsilon}} M^{\frac{\varepsilon-1}{\varepsilon}}}$. We are interested in how shocks to D and S affect the wage bill and value-added of the firm, which are given by:

$$E^L = \left(\frac{1}{\kappa} \right)^{\frac{1}{\gamma}} (L + L_f)^{\frac{\gamma+1}{\gamma}} \tag{C.9}$$

$$VA = D^{\frac{1}{\sigma}} \left[X(L,M) \right]^{\frac{\sigma-1}{\sigma}} - (M + M_f)/S \tag{C.10}$$

Note that given D and S, equations (C.7)-(C.10) implicitly define L, M, E^L, and VA. Hence, we can differentiate this system and solve for $\hat{L}$, $\hat{M}$, $\hat{E}^L$, and $\hat{VA}$ as functions of $\hat{D}$ and $\hat{S}$ (where $\hat{Y} \equiv d\log Y$ for any variable Y). Let β_y^x denote the pass-through of a shock to y to outcome x, so that we can write:

$$\hat{E}^L = \beta_D^{EL}\hat{D} + \beta_S^{EL}\hat{S} \tag{C.11}$$

$$\hat{VA} = \beta_D^{VA}\hat{D} + \beta_S^{VA}\hat{S} \tag{C.12}$$

We are interested in comparing the relative pass-through coefficients $\beta_D^{EL/VA} \equiv \beta_D^{EL}/\beta_D^{VA}$ and $\beta_S^{EL/VA} \equiv \beta_S^{EL}/\beta_S^{VA}$. Define $h_L \equiv L/\bar{L}$, $h_M \equiv M/\bar{M}$, $\rho \equiv \frac{VA}{VA+E^M}$, and $s \equiv \frac{E^M}{\frac{1}{\eta}E^L+E^M}$ (where $E^M \equiv \bar{M}/S$ is expenditure on materials). After a lot of tedious algebra, we can show that:

$$\beta_D^{EL/VA} = \frac{(1+\gamma)\,h_L}{(1+\gamma)\,h_L + \chi_D} \tag{C.13}$$

$$\beta_S^{EL/VA} = \frac{(1+\gamma)\,h_L}{(1+\gamma)\,h_L + \chi_S} \tag{C.14}$$

where:

$$\chi_D = \frac{1}{\mu h_M \rho}\left[\gamma\left[\mu h_M\left(1-h_L\right) + s\left(h_L - h_M\right)\right] + s h_L\left[h_M\left(\mu-1\right)\left(\varepsilon-1\right) + 1 - h_M\right]\right] \tag{C.15}$$

$$\chi_S = \frac{1}{\mu h_M \rho}\left[\gamma\left[\frac{1}{\sigma-\varepsilon} - \frac{h_M}{\sigma-1} + \mu h_M\left(1-h_L\right) + s\left(h_L-h_M\right)\right] + h_L\left[\frac{\sigma}{\sigma-\varepsilon} - \mu h_M\right]\right] \tag{C.16}$$

Note that to compare $\beta_D^{EL/VA}$ and $\beta_S^{EL/VA}$, we only need to compare χ_D and χ_S. The difference between the latter two terms can be expressed as:

$$\chi_D - \chi_S = \left[\frac{\gamma + h_L\left[\varepsilon s + \sigma\left(1-s\right)\right]}{\mu h_M \rho}\right]\left[\frac{\varepsilon^* - \varepsilon}{\left(\sigma-\varepsilon\right)\left(\sigma-\varepsilon^*\right)}\right] \tag{C.17}$$

where $\varepsilon^* \equiv 1 - (\sigma-1)\left(\frac{1-h_M}{h_M}\right)$. Since $\varepsilon^* < \sigma$, the sign of $\chi_D - \chi_S$ only depends on the sign of $\frac{\varepsilon-\varepsilon^*}{\sigma-\varepsilon}$. This lets us easily characterize the comparison of $\beta_D^{EL/VA}$ and $\beta_S^{EL/VA}$ as follows.

PROPOSITION **4.** *In the model with fixed costs, wage bill pass-through relative to value-added pass-through is (i) greater under demand shocks than supply shocks ($\beta_D^{EL/VA} > \beta_S^{EL/VA}$) when $\varepsilon \in (\varepsilon^*, \sigma)$, (ii) smaller under demand shocks than supply shocks ($\beta_D^{EL/VA} < \beta_S^{EL/VA}$) when $\varepsilon \in (0, \varepsilon^*) \cup (\sigma, \infty)$, and (iii) the same under demand and supply shocks ($\beta_D^{EL/VA} = \beta_S^{EL/VA}$) when $\varepsilon = \varepsilon^*$.*

Note that if there are no fixed material costs, then $h_M = 1$ and $\varepsilon^* = 1$. In this case, even in the presence of fixed labor costs, assuming Cobb-Douglas technology ($\varepsilon = 1$) would imply exactly the same pass-through of demand and supply shocks into worker earnings. In this sense, fixed labor costs are neither necessary nor sufficient to generate the asymmetric pass-through described in Fact 3.

D A Shapley Value Approach for Counterfactuals

In the counterfactual exercises studied in section 7, we deal with interdependencies between sources of variation in shaping inequality outcomes using the following approach. Let Θ denote the estimated vector of values for all model primitives and let $X(\Theta)$ denote the value of some equilibrium outcome X under this parameter vector. Now, define some N subsets of the parameter vector $\{\theta_n\}_{n=1}^N$ such that $\Theta = \cup_{n=1}^N \theta_n$ and denote $\mathcal{N} \equiv \{1, \cdots, N\}$. We are interested in computing values of outcome X under known counterfactual values $\hat{\theta}_n$ for each subset of the parameter vector. Therefore, let $\hat{\Theta}_S \equiv \left\{\cup_{n\in S}\hat{\theta}_n\right\} \cup \{\cup_{n\notin S}\theta_n\}$ denote the parameter vector under counterfactual values for parameter subsets in S for some $S \subseteq \mathcal{N}$. We define the Shapley value X_n for parameter subset n in relation to outcome X as follows:

$$X_n = \sum_{S\subseteq\mathcal{N}\setminus\{n\}} \frac{|S|!\,(N! - |S|! - 1)}{N!} \left[X\left(\hat{\Theta}_{S\cup\{n\}}\right) - X\left(\hat{\Theta}_S\right)\right] \tag{D.1}$$

For example, suppose that X is the variance of log earnings across all workers, θ_n is the estimated vector of firm TFPs, and $\hat{\theta}_n$ is a counterfactual vector of firm TFPs with each value equal to the mean of θ_n across firms. Then, we measure the contribution of TFP heterogeneity to earnings variance as $-\frac{X_n}{X(\Theta)}$. By construction of the Shapley value, these measures sum to one across all $n \in \mathcal{N}$.

E Robustness of Empirical Results

E.1 Stylized Facts After Controlling for Firm Size

We explore the role of firm size for the stylized facts through the lens of the model. For the relationship between the earnings premia W and upstream access S, the model predicts the following in log changes:

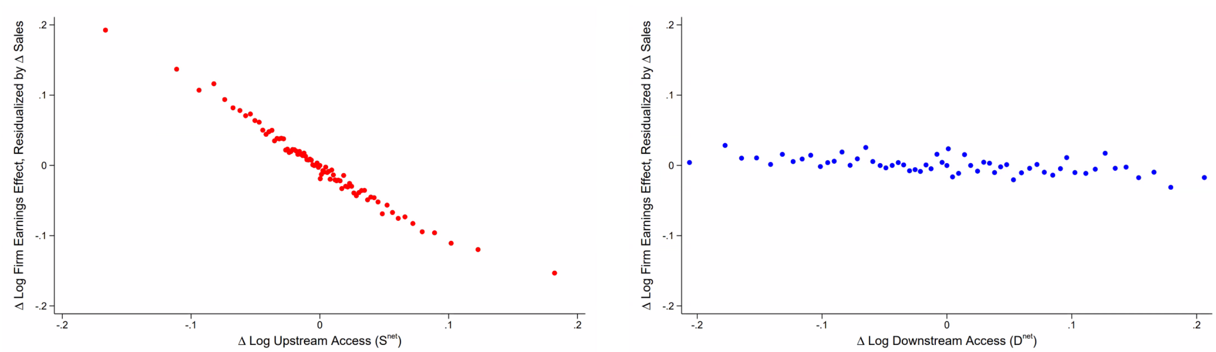
$$\hat{W} \;=\; \frac{1}{1+\gamma+(\varepsilon-1)s_M}\,\hat{R} \;-\; \frac{(\varepsilon-1)s_M}{1+\gamma+(\varepsilon-1)s_M}\,\hat{S} \tag{E.1}$$

where γ is the labor supply elasticity and $s_M \in (0,1)$ is the share of materials in total input costs. Under the restriction that $\varepsilon > 1$, the firm earnings premium is decreasing in upstream access conditional on sales. This is a pure substitution effect: controlling for changes in sales effectively controls for the scale effect associated with better access to suppliers, so all that is left is the substitution effect, which is negative if labor and materials are substitutes.

We take equation (E.1) to the data by correlating firms' earnings premia after residualizing firm size according to the model with upstream and downstream access. In particular, we correlate $\hat{W} - \frac{1}{1+\gamma+(\varepsilon-1)s_M}\hat{R}$ with $\frac{(\varepsilon-1)s_M}{1+\gamma+(\varepsilon-1)s_M}\hat{S}$ and $\hat{D}$. Figure A.I shows that, after controlling for firm size as the model implies, there is a negative correlation between earnings premia W and upstream access S and there is no correlation with downstream access D. Furthermore, the magnitudes of these correlations are consistent with the model's prediction. The correlation of $\hat{W} - \frac{1}{1+\gamma+(\varepsilon-1)s_M}\hat{R}$ with $\frac{(\varepsilon-1)s_M}{1+\gamma+(\varepsilon-1)s_M}\hat{S}$ is close to -1. This is yet another validation that $\epsilon > 1$.

For the relationship between the earnings premia W and downstream access D, note that downstream access does not enter the right-hand side of the above equation. This is because changes in demand only have scale effects, which are fully captured by changes in sales. This implies that after controlling for sales and upstream access, there should be no relationship between the firm earnings premium and downstream access. Consistent with this implication from the model, we find that the correlation between $\hat{W} - \frac{1}{1+\gamma+(\varepsilon-1)s_M}\hat{R}$ and $\hat{D}$ is close to zero.

Figure A.I: Firm earnings premia and network access, after residualizing by sales



Notes: All plots are generated using the bin scatter program provided by Michael Stepner: https://michaelstepner.com/software. The firm earnings effect is measured as f_{it} from the worker earnings decomposition in equation (3.1). The network access measures D^{net} and S^{net} are as defined in equation (3.3). The y-axis variable is, according to the model, $\hat{W} - \frac{1}{1+\gamma+(\varepsilon-1)s_M}\hat{R}$. In the left-hand-side graph, the x-axis variable is $\frac{(\varepsilon-1)s_M}{1+\gamma+(\varepsilon-1)s_M}\hat{S}$ where is in the right-hand-side graph it is $\hat{D}$. Changes are implemented as 5-year changes between 2005 and 2010. We fix s_M as the average value per firm over the 2005-2010 period. The sample used for this graph is a subset of the firm baseline described in Section 2. It is a subset given that we take 5-year differences. Thus, it corresponds to the same sample as the one used for the pass-through reduced-form analysis. All variables are parsed of industry-municipality-year means.

For the relationship between labor share of cost $s_{L/C}$, sales R, downstream access D, and upstream access S, the labor share of cost can be expressed as:

$$s_{L/C} = 1 - \left[1 + \eta\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{WS}{\omega}\right)^{1-\epsilon}\right]^{-1}$$

Thus, in log changes, the following holds:

$$\hat{s}_{L/C} \;=\; \frac{(1-\varepsilon)s_M}{1+\gamma+(\varepsilon-1)s_M}\,\hat{R} \;-\; \frac{(1+\gamma)(\varepsilon-1)s_M}{1+\gamma+(\varepsilon-1)s_M}\,\hat{S} \tag{E.2}$$

The same results hold for the labor share of value-added, since this varies monotonically with the labor share of cost.

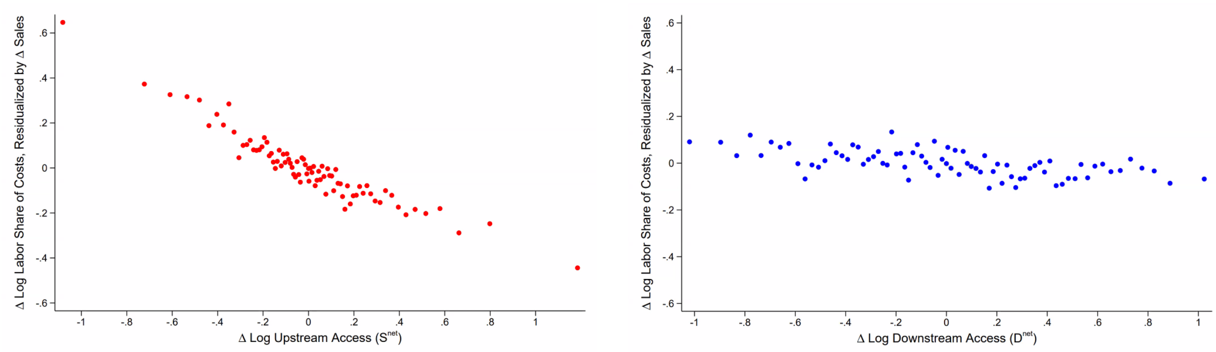
We take equation (E.2) to the data by correlating firms' labor share of cost after residualizing firm size according to the model with upstream and downstream access. In particular, we correlate $\hat{s}_{L/C} - \frac{(1-\varepsilon)s_M}{1+\gamma+(\varepsilon-1)s_M}\,\hat{R}$ with $\frac{(1+\gamma)(\varepsilon-1)s_M}{1+\gamma+(\varepsilon-1)s_M}\,\hat{S}$ and $\hat{D}$. Figure A.II shows that, after controlling for firm size as the model implies, there is a negative correlation between firms' labor share of cost and upstream access S and there is no correlation with downstream access D. Furthermore, the magnitudes of these correlations are consistent with the model's prediction. Furthemore, as shown in equation (E.2), a negative correlation between $\hat{s}_{L/C} - \frac{(1-\varepsilon)s_M}{1+\gamma+(\varepsilon-1)s_M}\,\hat{R}$ and $\frac{(1+\gamma)(\varepsilon-1)s_M}{1+\gamma+(\varepsilon-1)s_M}\,\hat{S}$ is yet another validation that $\epsilon > 1$.

For the relationship between firms' labor share of costs and downstream access D, note that downstream access does not enter the right-hand side of equation (E.2). This is because changes in demand only have scale effects, which are fully captured by changes in sales. This implies that after controlling for sales and upstream access, there should be no relationship between the firms' labor share of cost and downstream access. Consistent with this implication from the model, we find that the correlation between $\hat{s}_{L/C} - \frac{(1-\varepsilon)s_M}{1+\gamma+(\varepsilon-1)s_M}\,\hat{R}$ and $\hat{D}$ is close to zero.

E.2 Sectoral Differences in the Labor-Materials Substitution Elasticity

We explore sectoral differences in the elasticity of substitution between labor and materials. We implement the same design described in Section 6.2.5 sector by sector. To ensure that we had enough statistical power, we aggregated all sectors into 4 groups: primary sectors

Figure A.II: Labor cost shares and network access, after residualizing by sales



Notes: All plots are generated using the bin scatter program provided by Michael Stepner: https://michaelstepner.com/software. The network access measures D^{net} and S^{net} are as defined in equation (3.3). The y-axis variable is, according to the model, $\hat{s}_{L/C} - \frac{(1-\varepsilon)s_M}{1+\gamma+(\varepsilon-1)s_M}\hat{R}$. In the left-hand-side graph, the x-axis variable is $\frac{(1+\gamma)(\varepsilon-1)s_M}{1+\gamma+(\varepsilon-1)s_M}\hat{S}$ where is in the right-hand-side graph it is $\hat{D}$. Changes are implemented as 5-year changes between 2005 and 2010. We fix s_M as the average value per firm over the 2005-2010 period. The sample used for this graph is the same as the one used for the pass-through reduced-form analysis. All variables are parsed of industry-municipality-year means.

(which includes agriculture, fishing and mining), manufacturing (which includes also utilities and construction), trade (which includes retail, wholesale, transportation and telecommunications), and other services (which includes financial, professional and social services). Table A.III shows that all sectors exhibit elasticities of substitution between labor and materials larger than 1. Furthermore, the elasticity is largest in the manufacturing sector and lowest in services.

Table A.III: Sectoral Estimation of labor-materials substitution elasticity, ϵ

	$\log E^M/E^L$				
	(1) All Sectors	(2) Primary	(3) Manufacturing	(4) Trade	(5) Services
$\log W/Z$	0.553	0.253	0.726	0.406	0.164
	(0.058)	0.090	(0.181)	(0.139)	(0.052)
ϵ	1.55	1.25	1.73	1.41	1.16
Model for Wage Component	BLM	BLM	BLM	BLM	BLM
Instruments	$\{E^M_{it-1}, E^L_{it-1}\}$	$\{E^M_{it-1}, E^L_{it-1}\}$	$\{E^M_{it-1}, E^L_{it-1}\}$	$\{E^M_{it-1}, E^L_{it-1}\}$	$\{E^M_{it-1}, E^L_{it-1}\}$
Instrument Polynomial	Quadratic	Quadratic	Quadratic	Quadratic	Quadratic
First Stage F-Stat	130	23.2	21.1	27	6
Hansen's J Test	0.121	0.432	0.122	0.254	0.122
Number of Observations	44,967	7,566	13,628	17,680	6,093

Notes: This table presents estimates of equation (6.10) using the baseline firm-level dataset sector by sector. Sectors are aggregated into 4 groups: primary sectors (which includes agriculture, fishing and mining), manufacturing (which includes also utilities and construction), trade sectors (which includes retail, wholesale, transportation and telecommunications), and other services (which includes financial, professional and social services). Column 1 repeats the economy-wide elasticity presented in Table 3. Columns 2-5 presents the result for each of the 4 sectors. All specifications are estimated using two-stage GMM with a robust weighting matrix. Standard errors are shown in parentheses.

Online Appendix to

"Earnings Inequality in Production Networks"

by

Federico Huneeus, Kory Kroft, and Kevin Lim

A Earnings Variance Decomposition

Following Lamadon et al. (2022), we first utilize equation (3.1) to decompose the variance of log earnings as:

$$\text{var}\left(\log w_{imt}\right) = \underbrace{\text{var}\left(\tilde{x}_m\right)}_{57\%} + \underbrace{\text{var}\left(\log \tilde{f}_{it}\right)}_{10.8\%} + \underbrace{2\text{cov}\left(\tilde{x}_m, \log \tilde{f}_{it}\right)}_{19.8\%} + \underbrace{int}_{\text{-2.0\%}} + \underbrace{\text{var}\left(\hat{x}_{mt}\right)}_{14.4\%} \tag{A.1}$$

where $\tilde{x}_m \equiv (x_m - \bar{x})\,\bar{\theta}$ is the worker effect when employed at the average firm, $\log \tilde{f}_{it} \equiv \log f_{it} + \theta_i \bar{x}$ is the firm effect when matched with the average worker, $\left\{\bar{x}, \bar{\theta}\right\}$ denote the averages of $\{x_m, \theta_i\}$ across workers, *int* collects terms arising from non-linear interactions between the worker and firm effects:

$$int \equiv \text{var}\left[\left(\theta_i - \bar{\theta}\right)(x_m - \bar{x})\right] + 2\text{cov}\left[\tilde{x}_m + \log \tilde{f}_{it}, \left(\theta_i - \bar{\theta}\right)(x_m - \bar{x})\right] \tag{A.2}$$

and all variances and covariances are computed at the worker-level. Unsurprisingly, we find that the variance of $\tilde{x}_m$ accounts for the majority of earnings variance. However, firms also play an important role: the variance of the $\log \tilde{f}_{it}$ accounts for 10.8% of log earnings variance, while the sorting covariance between $\tilde{x}_m$ and $\log \tilde{f}_{it}$ explains 19.8%.

B Model Extension with Capital Inputs

Suppose that firms produce output using capital in addition to labor and materials with a production function of the following form:

$$X_{it} = T_{it} K_{it}^{\alpha} F\left[\left\{\phi_{it}(a)\, L_{it}(a), M_{it}(a)\right\}_{a \in A}\right]^{1-\alpha} \tag{B.1}$$

where α is the capital share of cost. Suppose also that capital is available at a price r_{it} that may vary across firms due to differences in access to capital markets. The firm's profit maximization problem can now be written as:

$$\max_{K_{it}, \{w_{it}(a), M_{it}(a)\}_{a \in A}} \left\{ D_{it}^{\frac{1}{\sigma}} X_{it}^{\frac{\sigma-1}{\sigma}} - \sum_{a \in A} w_{it}(a)\, L_{it}(a) - \frac{1}{S_{it}} \sum_{a \in A} M_{it}(a) - r_{it} K_{it} \right\} \tag{B.2}$$

subject to the production function (B.1) and labor supply curves, $L_{it}(a) = \kappa_{it}(a)\, w_{it}(a)^{\gamma}$. The first-order condition for this problem with respect to the capital input is:

$$\alpha D_{it}^{\frac{1}{\sigma}} X_{it}^{\frac{\sigma-1}{\sigma}} = r_{it} K_{it} \tag{B.3}$$

Using this to substitute for the choice of capital, we can rewrite the profit maximization problem as a choice over wages and material inputs alone, as in the original problem:

$$\max_{\{w_{it}(a),M_{it}(a)\}_{a\in A}} \left\{ D_{it}^{\frac{1}{\tilde{\sigma}}} \tilde{X}_{it}^{\frac{\tilde{\sigma}-1}{\tilde{\sigma}}} - \sum_{a\in A} w_{it}(a) L_{it}(a) - \frac{1}{S_{it}} \sum_{a\in A} M_{it}(a) \right\} \tag{B.4}$$

$$\text{s.t. } \tilde{X}_{it} = \tilde{T}_{it} F\left[\{\phi_{it}(a) L_{it}(a), M_{it}(a)\}_{a\in A} \right] \tag{B.5}$$

where $\tilde{\sigma} \equiv \sigma(1-\alpha)+\alpha$ and $\tilde{T}_{it} \equiv (1-\alpha)^{\frac{\tilde{\sigma}}{\tilde{\sigma}-1}} \left(\frac{\alpha}{r_{it}}\right)^{\frac{\alpha}{1-\alpha}} T_{it}^{\frac{1}{1-\alpha}}$. Hence, the firm's problem with capital is isomorphic to the problem without capital if one replaces σ with $\tilde{\sigma}$ and T_{it} with $\tilde{T}_{it}$. Note that the introduction of capital lowers the effective price elasticity of demand (since $\tilde{\sigma} < \sigma$ for any $\alpha \in (0,1)$), while differences in capital prices r_{it} can be viewed as differences in effective productivity.

C Cobb-Douglas and Value-added Production Functions

We examine here a special case of the model where production technologies are of a Cobb-Douglas form ($\epsilon \to 1$). This case admits a closed-form solution for the firm earnings premium and hence is useful for providing a more transparent discussion of key mechanisms. Under Cobb-Douglas technology, the firm's profit maximization problem can be rewritten by first solving out for the optimal choice of material inputs:

$$\max_{\{w_{it}(a)\}_{a\in A}} \left\{ A_{it}\tilde{X}_{it}^{1-\alpha} - \sum_{a\in A} w_{it}(a) L_{it}(a) \right\} \tag{C.1}$$

$$\text{s.t. } \tilde{X}_{it} = \sum_{a\in A} \phi_{it}(a) L_{it}(a) \tag{C.2}$$

where $A_{it}\tilde{X}_{it}^{1-\alpha}$ is equal to nominal value-added for firm i, $\alpha \equiv \frac{1}{\sigma\lambda+(1-\lambda)} > 0$ reflects curvature in value-added arising from imperfectly elastic demand ($\sigma < \infty$), and A_{it} is a composite term that can be interpreted as *value-added productivity*:

$$A_{it} \equiv \text{const.} \times T_{it}^{\frac{\sigma-1}{\sigma\lambda+1-\lambda}} \omega_{it}^{\frac{\lambda(\sigma-1)}{\sigma\lambda+1-\lambda}} D_{it}^{\frac{1}{\sigma\lambda+1-\lambda}} S_{it}^{\frac{(1-\lambda)(\sigma-1)}{\sigma\lambda+1-\lambda}} \tag{C.3}$$

Equations (C.1) and (C.2) represent the firm's profit maximization problem in terms of a value-added production function, which is a common approach in the literature (see Lamadon et al. (2022), for example). The firm earnings preium W_{it} can then be solved for explicitly as:

$$W_{it} = \text{const.} \times A_{it}^{\frac{\sigma\lambda+1-\lambda}{\gamma+\sigma\lambda+1-\lambda}} \left(\bar{\phi}_{it}\right)^{-\frac{1}{\gamma+\sigma\lambda+1-\lambda}} \tag{C.4}$$

Hence, in this special case of the model, demand and supply shocks in the network that operate through $\{D_{it}, S_{it}\}$ act as shifters of value-added productivity A_{it}. The introduction of production networks therefore provides a microfoundation for value-added productivity. In particular, from equation (C.4), it is clear that increases in demand D_{it} and supplier efficiency S_{it} lead to increases in the earnings premium W_{it}.

However, several points are worth noting. First, identification of A_{it} alone does not allow one to separately identify the components of A_{it} (and hence of the firm earnings premium W_{it}) that stem from TFP, labor productivity, and network characteristics. Hence, the value-added approach leaves open the question of how heterogeneity in production network linkages shapes earnings inequality. Second, the value-added representation of the firm's production function is only valid when $\epsilon = 1$ (unless output markets are perfectly competitive so that $\sigma \to \infty$, in which case the value-added representation is valid for any ϵ). When this condition does not hold, the concept of value-added productivity is no longer meaningful. Third, as discussed in the paper, we estimate ϵ and find that it is statistically greater than 1. Finally, the assumption of Cobb-Douglas technology leads to two counterfactual predictions: it restricts labor shares of cost to be constant across firms and, as discussed in the paper, it implies complete pass-through of changes in firm value-added per worker to changes in worker earnings, which is at odds with existing empirical evidence (see Card et al. (2018), Kline et al. (2019), and Berger et al. (2022), for example). Hence, while the Cobb-Douglas case is useful as a heuristic for developing the intuition behind the model, it is a simplification that is unsupported by our data.

D Dependency of pass-through on the labor-materials substitution elasticity

Here, we consider shocks to a firm's TFP, demand shifter, or inverse unit cost of materials, $X \in \left\{T, D, S \equiv \frac{1}{Z}\right\}$. We are interested in the relative pass-through $\beta^X \equiv \frac{\partial \log E_{it}^L}{\partial \log X_{it}} / \frac{\partial \log VA_{it}}{\partial \log X_{it}}$ into a firm's wage bill E_{it}^L versus its value-added VA_{it}. As shown in the proof of Proposition 3, these relative pass-through coefficients depend only on a firm's material share of cost and the structural elasticities $\{\gamma, \sigma, \epsilon\}$. Figure 1 shows how these relative pass-through coefficients vary with ϵ, setting the material cost share to the median value across firms in our data and the elasticities $\{\gamma, \sigma\}$ to the corresponding values obtained from our structural estimation of the model. Note that pass-through is incomplete if and only if $\epsilon > 1$. Furthermore, demand-driven and TFP-driven growth benefit workers more than cost-driven growth if and only if $\epsilon > 1$. The same results hold for sales instead of VA or pass-through into wages vs. value-added per worker.

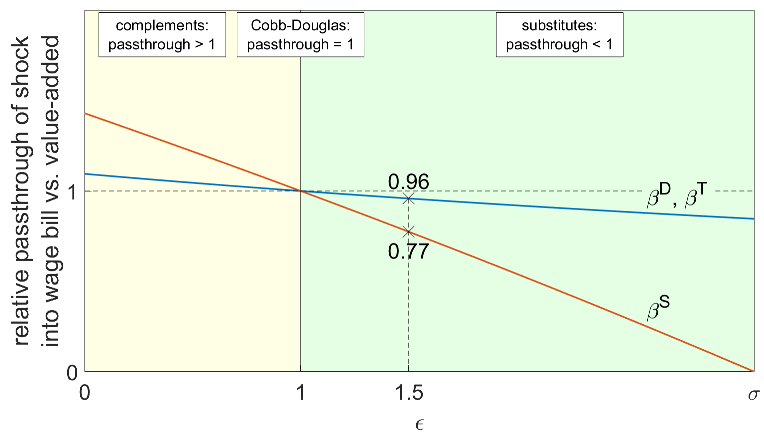
E Details of the Structural Estimation of the Model

E.1 Labor supply elasticity

We first formally describe the identification of γ in the presence of measurement error in wage bills. To this end, suppose that wage bills in the data $\ddot{E}_{it}^L$ are related to wage bills in the model E_{it}^L as follows:

$$\log E_{it}^L = \log \ddot{E}_{it}^L + e_{it}^L \tag{E.1}$$

Figure 1: Dependence of pass-through on the labor-materials substitution elasticity



Notes: This figure shows how the labor-materials substitution elasticity ϵ matters for the relative pass-through into a firm's wage bill versus its value-added following a TFP shock (β^T), downstream demand shock (β^D), or upstream efficiency shock (β^S). The figure also shows the relative pass-through coefficients corresponding to our estimated value of $\epsilon = 1.5$.

where e^L_{it} denotes an MA(k) measurement error given by $e^L_{it} = \sum_{s=0}^{k} \delta^{L,s} u^L_{i,t-s}$ for some weights $\delta^{L,s}$ and mean-zero shocks u^L_{it} that are iid across firms and time. In this case, equation (6.4) becomes:

$$\Delta \log w_{imt} = \frac{1}{1+\gamma} \Delta \log \ddot{E}^L_{it} + \Delta \log \hat{a}_{mt} + \frac{1}{1+\gamma} \Delta e^L_{it} \tag{E.2}$$

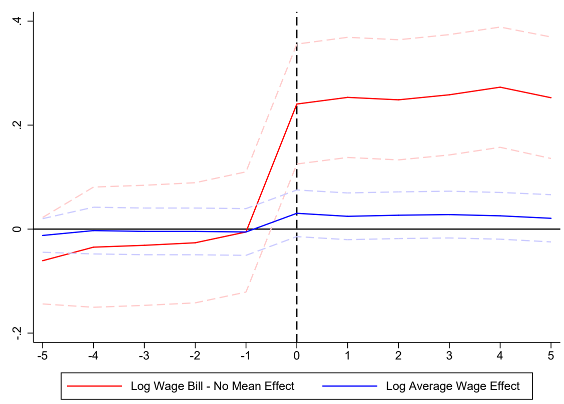
In the absence of measurement error, the residual in equation (E.2) contains only worker-level shocks ($\Delta \log \hat{a}_{mt}$), which are orthogonal to changes in firm wage bills under Assumption 6.5. However, with measurement error in wage bills, the unobserved error term in equation (E.2) contains a component that is potentially correlated with observed changes in the wage bill.

To address this, note that Δe^L_{it} depends only on measurement error shocks u^L_{it} in periods $\{t-k-1, \cdots, t\}$. Hence, as long as u^L_{it} is orthogonal to all *lagged* innovations in the Markov processes for time-varying firm primitives, lagged changes in wage bills $\log \Delta \ddot{E}^L_{is}$ for any $s < t-k-1$ are valid instruments for $\log \Delta \ddot{E}^L_{it}$ in identifying γ from equation (E.2). The relevance of these instruments requires serial correlation in $\Delta \ddot{E}^L_{it}$ to be non-zero with at least $k+2$ lags, which is also consistent with the Markov processes for firm productivities specified in Assumption 6.4.

For robustness, we also follow Lamadon et al. (2022) and estimate γ using a difference-in-difference approach (DiD). For this, we follow a three step procedure. First, for each year, we order firms according to log changes of the wage bill of the firm. Second, we identify the treatment when firms have log changes of their wage bill above the median of log changes of wage bill across firms each year. Finally, we plot difference in wage bill of treated and control firms both at each year ($t = 0$) and years before ($t < 0$) and after ($t > 0$). We perform this step for each calendar year and weight firms by the number of workers.

Results are presented Figure 2. By construction, the treatment and control groups differ in the wage bill from period $t = -1$ to $t = 0$. On average, firms in the treatment group face an increase of 21 log points growth in their wage bill relative to firms in the control group. The effect of the treatment appears to be permanent in levels up to 5 years after the treatment. Figure 2 also shows the effect on the average earnings of firms. On average, firms in the treatment group face an increase of 3.25 log points of their average earnings relative to firms in the control group. Once again, the effect of the treatment appears to be permanent in levels up to 5 years after the treatment. Finally, firms in the treatment and control group do not experience statistically significant differences up to 5 years before the treatment, for both the wage bill and the average earnings. Through the lens of a DiD design, these results imply a pass-through rate of firms shocks of around 0.155 (= 0.0325/0.21). From equation (6.3), this implies a labor supply elasticity of $\hat{\gamma} = 5.5$, which is the same as our preferred estimate documented in the main text.

Figure 2: Difference-in-difference Estimate of pass-through of Firm Shocks to Worker Earnings



Notes: This figure presents the results from the Lamadon et al. (2022) difference-in-difference approach to estimating pass-through of wage bill shocks to worker wages.

E.2 Worker and firm wage effects

To estimate the Bonhomme et al. (2019) decomposition of worker earnings from equation (6.5), we first cluster firms using a k-means clustering algorithm into $K = 10$ groups. We use a weighted K-means algorithm with 100 randomly generated starting values. We use firms' empirical distributions of log earnings on a grid of 10 percentiles of the overall log-earnings distribution. Second, we use these K groups as the relevant firm identifier in the Bonhomme et al. (2019) estimation approach. This procedure yields estimates of the firm fixed effect $\bar{W}_k$ and the worker-firm production complementarity parameter θ_k for every firm cluster k, as well as the permanent and transient components of ability for every worker. Our estimates of $\log \bar{W}_k$ and θ_k obtained using this procedure are presented in Table 1, where clusters are sorted according to the former variable.

To assess robustness of our results to the number of clusters used, Table 2 documents the share of variance of wages accounted for by the firm fixed effect $\bar{W}_i$. We implement this for the basic model of Abowd et al. (1999) and also the basic version of the model of Bonhomme et al. (2019) with only firm and worker fixed effects for different levels of

Table 1: Estimates of firm fixed effects and production complementarities

Cluster	1	2	3	4	5	6	7	8	9	10
$\log \bar{W}_k$	0	0.25	0.61	0.89	1.06	1.24	1.50	1.69	1.80	1.92
θ_k	1	1.13	1.42	1.66	1.77	1.91	2.19	2.37	2.44	2.26

Notes: This table presents estimates of firm fixed effects $\log \bar{W}_k$ and production complementarities θ_k in the earnings equation (6.5) by earnings cluster k using the movers sample. Clusters are sorted in ascending order of $\log \bar{W}_k$. Note that $\log \bar{W}_k$ and θ_k are normalized to zero and one respectively for firms in the first earnings cluster.

K (thus, excluding interactions and time-varying firm effects). First, one can see that the basic version of the model of Bonhomme et al. (2019) implies a role for the firm fixed effect that is significantly lower than the model of Abowd et al. (1999), consistent with previous literature that has found that addressing the limited mobility bias inherent in estimates of Abowd et al. (1999) decreases the share of the variance accounted for by the firm fixed effect (Bonhomme et al., 2020). Second, as one increases K from 10 to 50, the share of the variance of wages accounted for the firm fixed effects increases only 0.7 percentage points from 7.8 to 8.5%. At least with this piece of evidence, this implies that the limited mobility bias does not represent a substantially bigger problem for $K = 50$ than what it represents for $K = 10$.

Table 2: Share of Log Earnings Variance Accounted for by the Firm Fixed Effect

Estimation Strategy	Number of Clusters	Firm Fixed EffectShare
AKM		12.3
BLM	10	7.8
BLM	50	8.5

Notes: This table documents the share of the log of earnings variance accounted for by the firm fixed effect. It is documented for the estimation strategy of Abowd et al. (1999) (row 1), for the estimation strategy of Bonhomme et al. (2019) with $K = 10$ clusters (row 2) and the estimation strategy of Bonhomme et al. (2019) with $K = 50$ clusters (row 3).

To further assess whether clustering with $K = 10$ or $K = 50$ makes a difference, we document how much clusters account for the variance of firm-level characteristics. Tables 3-4 document the share of the variance of variables accounted for by within-cluster variation. Table 3 shows the within-cluster share of variance of variables in levels, whereas Table 4 shows the same evidence for variables in ratios. Although there is substantial heterogeneity across firms that the clustering procedure of Bonhomme et al. (2019) does not account for, this result does not vary significantly if one uses $K = 10$ or $K = 50$ clusters.

Table 3: Within Clusters Share of Total Variance of Variables in Levels

Number Clusters	Total Sales	Materials	Wage Bill	Employment	Number of Buyers	Number of Suppliers	Firm-to-Firm Sales
10	79	90	67	88	90	85	95
50	74	86	62	84	88	81	92

Notes: This table documents the share of the variance of each variable accounted for by the within cluster variance. It is implemented for $K = 10$ and $K = 50$ and for variables in levels.

Table 4: Within Clusters Share of Total Variance of Variables in Ratios

Number Clusters	Wage Bill/Sales	Materials/Sales	Materials/Wage Bill	Sales/Employment	Wage Bill/Employment	Materials/Employment
10	96	97	95	92	26	99
50	95	97	95	90	21	98

Notes: This table documents the share of the variance of each variable accounted for by the within cluster variance. It is implemented for $K = 10$ and $K = 50$ and for variables in ratios.

E.3 Amenities

To estimate firm amenities, we begin with the labor supply equation (4.2). It will be useful for the exposition to write this explicitly in terms of permanent and transient worker abilities:

$$\frac{L_{it}\left(\bar{a},\hat{a}\right)}{L\left(\bar{a},\hat{a}\right)}=\frac{\left[g_i\left(\bar{a}\right)w_{it}\left(\bar{a},\hat{a}\right)\right]^{\gamma}}{\sum_{j\in\Omega^F}\left[g_j\left(\bar{a}\right)w_{jt}\left(\bar{a},\hat{a}\right)\right]^{\gamma}} \tag{E.3}$$

where note that under Assumption 6.1, amenity values only vary across workers in relation to permanent ability $\bar{a}$. Next, consider the equilibrium wage equation (4.14). Under assumption 6.1, we can write this as:

$$w_{it}\left(\bar{a},\hat{a}\right)=\eta\bar{a}^{\theta_i}\hat{a}W_{it} \tag{E.4}$$

The average wage paid by firm i to workers with permanent ability $\bar{a}$ is hence:

$$\bar{w}_{it}\left(\bar{a}\right)=\eta\bar{a}^{\theta_i}\mathbb{E}\left[\hat{a}\right]W_{it} \tag{E.5}$$

where $\mathbb{E}\left[\hat{a}\right]$ denotes the average value of transient ability. Under Assumptions 6.1 and 6.5, this mean does not depend on permanent ability of the worker or the identity of the firm. Combining (E.4) and (E.5), we then have:

$$w_{it}\left(\bar{a},\hat{a}\right)=\bar{w}_{it}\left(\bar{a}\right)\frac{\hat{a}}{\mathbb{E}\left[\hat{a}\right]} \tag{E.6}$$

Substituting this into (E.3) and using the decomposition of amenities in equation (6.11), we obtain:

$$\frac{L_{it}\left(\bar{a},\hat{a}\right)}{L\left(\bar{a},\hat{a}\right)}=\frac{\left[\tilde{g}_i\bar{g}_{k(i)}\left(\bar{a}\right)\bar{a}^{\theta_{k(i)}}W_{it}\right]^{\gamma}}{\sum_j\left[\tilde{g}_j\bar{g}_{k(j)}\left(\bar{a}\right)\bar{a}^{\theta_{k(j)}}W_{jt}\right]^{\gamma}} \tag{E.7}$$

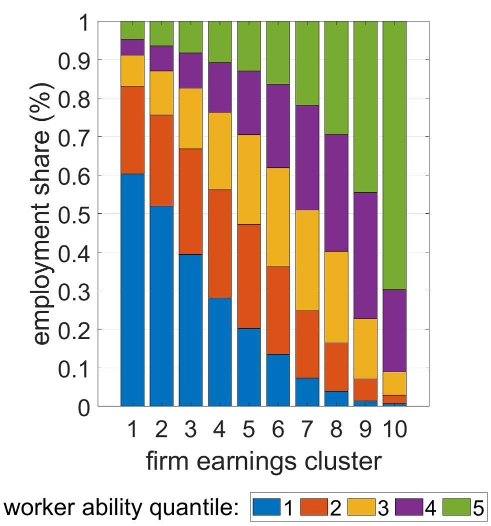
Now notice that the employment share of workers of ability $\{\bar{a},\hat{a}\}$ varies across firms only in relation to permanent ability $\bar{a}$. This is a direct implication of Assumption 6.1, which implies that workers do not sort to firms based on transient ability $\hat{a}$. Therefore, the share of workers of permanent ability $\bar{a}$ employed by firm i is also given by equation (E.7). Summing this (E.7) across all firms within cluster k, we can similarly express the share of workers of permanent ability $\bar{a}$ that are employed by firms in cluster k as:

$$\Lambda_{kt}\left(\bar{a}\right) = \frac{\sum_{i\in k}\left[\tilde{g}_i \bar{g}_k\left(\bar{a}\right)\bar{a}^{\theta_k} W_{it}\right]^{\gamma}}{\sum_j\left[\tilde{g}_j \bar{g}_{k(j)}\left(\bar{a}\right)\bar{a}^{\theta_{k(j)}} W_{jt}\right]^{\gamma}} \tag{E.8}$$

Next, note that for each value of permanent ability $\bar{a}$, equilibrium outcomes are invariant to scaling $g_i\left(\bar{a}\right)$ by a constant for all firms i. Therefore, we are allowed to choose one normalization of amenity values for each permanent worker ability type $\bar{a}$. For this, we choose $\sum_j\left[\tilde{g}_j \bar{g}_{k(j)}\left(\bar{a}\right)\bar{a}^{\theta_{k(j)}} W_{jt}\right]^{\gamma} = 1$. Furthermore, mean differences in amenity values can be loaded onto either $\tilde{g}_i$ or $\bar{g}_{k(i)}\left(\bar{a}\right)$. Hence, we are allowed to choose one normalization of the values for $\tilde{g}_i$ for each firm cluster. For this, we choose $\sum_{i\in k}\left[\tilde{g}_i W_{it}\right]^{\gamma} = 1$. With these normalizations, equation (6.12) follows immediately.

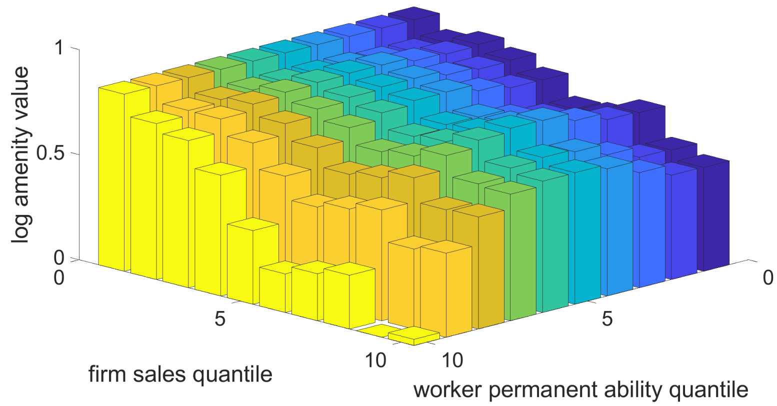
Our results are summarized in Figure 4, which shows average log amenity values by deciles of firm sales and worker permanent ability. Evidently, larger firms tend to offer lower amenity values to workers of each ability type, with this relationship being more pronounced for workers of higher permanent ability. Furthermore, as shown in Figure 3, our estimates of amenities and production complementarities imply positive sorting of workers to firms. Note that by construction, the model provides an exact fit to the cluster-level employment shares shown in the figure.

Figure 3: Employment shares by firm earnings cluster and worker ability



Notes: Firm earnings clusters are sorted in ascending order of the time-invariant firm earnings effect, $\bar{W}_{k(i)}$.

Figure 4: Distribution of Amenities



Notes: This figure shows the joint distribution of amenity estimates $\log g_i(\bar{a})$ by deciles of firm sales and worker permanent ability. Values are normalized for presentation purposes such that: (i) average log amenities within the smallest decile of firm sales are equal across deciles of worker permanent ability, and (ii) the smallest value of mean log amenities across sales-ability quantiles is equal to zero.

E.4 Firm relationship capability and relationship-specific productivity

To estimate equation (6.7), firms must have multiple connections. To identify seller fixed effects, each seller needs to have at least two buyers. Similarly, to identify buyer fixed effects, each buyer needs to have at least two sellers. In the data, some firms have either one supplier or one seller. Hence, we implement the aforementioned restriction using an iterative approach known as "avalanching". Specifically, we first drop firms with one supplier or seller. Doing this may result in additional firms that have one supplier or seller, hence in the next step, we drop these firms as well. We continue this process until firms are no longer dropped from the sample. The algorithm takes three iterations to converge in practice and reduces the sample size of firm-to-firm linkages from a total of 32 million transactions to 31.7 million transactions, that is, a reduction of 1% of transactions. Hence, the avalanching algorithm has little impact on our sample size. Bernard et al. (2022) report that avalanching also eliminates around 1% of firm-to-firm links in the production network for Belgium.

E.5 Labor-materials substitution elasticity and labor productivity

For estimation of ϵ using equation (6.10), we follow the approach in Doraszelski and Jaumandreu (2018). To control for $F^{\omega}(\omega_{i,t-1})$, we first rearrange the $t-1$ version of equation

(6.10) to write:

$$\log \omega_{i,t-1} = \frac{1}{\epsilon - 1} \log \left[\frac{1}{\eta} \left(\frac{1-\lambda}{\lambda} \right) \right] - \frac{1}{\epsilon - 1} \log \frac{E^M_{i,t-1}}{E^L_{i,t-1}} + \log \frac{W_{i,t-1}}{Z_{i,t-1}} \tag{E.9}$$

$$\equiv G\left(\log \frac{E^M_{i,t-1}}{E^L_{i,t-1}}, \log \frac{W_{i,t-1}}{Z_{i,t-1}} \right) \tag{E.10}$$

Substituting this into (6.10), we obtain:

$$\log \frac{E^M_{it}}{E^L_{it}} = \log \left[\frac{1}{\eta} \left(\frac{1-\lambda}{\lambda} \right) \right] + (\epsilon - 1) \log \frac{W_{it}}{Z_{it}} + H\left(\log \frac{E^M_{i,t-1}}{E^L_{i,t-1}}, \log \frac{W_{i,t-1}}{Z_{i,t-1}} \right) \tag{E.11}$$

$$+ (1-\epsilon)\, \xi^\omega_{it} \tag{E.12}$$

where $H(\cdot,\cdot) \equiv (1-\epsilon) F^\omega [G(\cdot,\cdot)]$. Hence, we control for the term H using polynomials in lagged relative expenditures $\log \frac{\tilde{E}^M_{i,t-1}}{\tilde{E}^L_{i,t-1}}$ and lagged relative input prices $\log \frac{\tilde{W}_{i,t-1}}{Z_{i,t-1}}$. In addition, we follow Doraszelski and Jaumandreu (2018) and instrument for relative input prices at date t using polynomials in one-period lags of logged input expenditures and factor prices.

E.6 Firm TFP

We choose values for TFP T_{it} to fit the estimated firm earnings premia W_{it}. We do this using an iterative numerical procedure that is similar in spirit to the equilibrium solution algorithm described below in section F:

1. Compute $\left\{\bar{\phi}_{it}\right\}_{i\in\Omega^F}$ from (4.18), using (4.3), (4.4), and the estimated firm earnings premia $\{W_{it}\}_{i\in\Omega^F}$.
2. Guess E_t.
   (a) Guess $\{D_{it}, S_{it}\}_{i\in\Omega^F}$.
   (b) Compute the values of $\{T_{it}\}_{i\in\Omega^F}$ implied by equation (F.1) below, given the estimated firm earnings premia $\{W_{it}\}_{i\in\Omega^F}$.
   (c) Compute new guesses of $\{D_{it}\}_{i\in\Omega^F}$ and $\{S_{it}\}_{i\in\Omega^F}$ from (4.10).
   (d) Iterate on steps (a)-(c) until convergence.
3. Compute a new guess of E_t from (4.20), using (4.2), (4.12), and (4.14).
4. Iterate on steps 1-2 until convergence.

F Numerical Solution Algorithm

We solve numerically for an equilibrium of the model using the following solution algorithm.

1. Guess E_t.

(a) Guess $\left\{\Delta_{it}, \Phi_{it}, \bar{\phi}_{it}\right\}_{i\in\Omega^F}$.

(b) Compute $\{D_{it}\}_{i\in\Omega^F}$ and $\{S_{it}\}_{i\in\Omega^F}$ from (4.10).

(c) Solve for $\{W_{it}, \nu_{it}, X_{it}\}_{i\in\Omega^F}$ from (4.13), (4.15), and (4.16).

(d) Compute new guesses of $\{\Delta_{it}\}_{i\in\Omega^F}$ and $\{\Phi_{it}\}_{i\in\Omega^F}$ from (4.11) and $\left\{\bar{\phi}_{it}\right\}_{i\in\Omega^F}$ from (4.18).

(e) Iterate on steps (a)-(d) until convergence.

2. Compute a new guess of E_t from (4.20), using (4.2), (4.12), and (4.14).

3. Iterate on steps 1-2 until convergence.

Note that step 1(c) involves numerical solution of a system in $\{W_{it}, \nu_{it}, X_{it}\}$. This system can be reduced to one in the firm earnings premium alone:

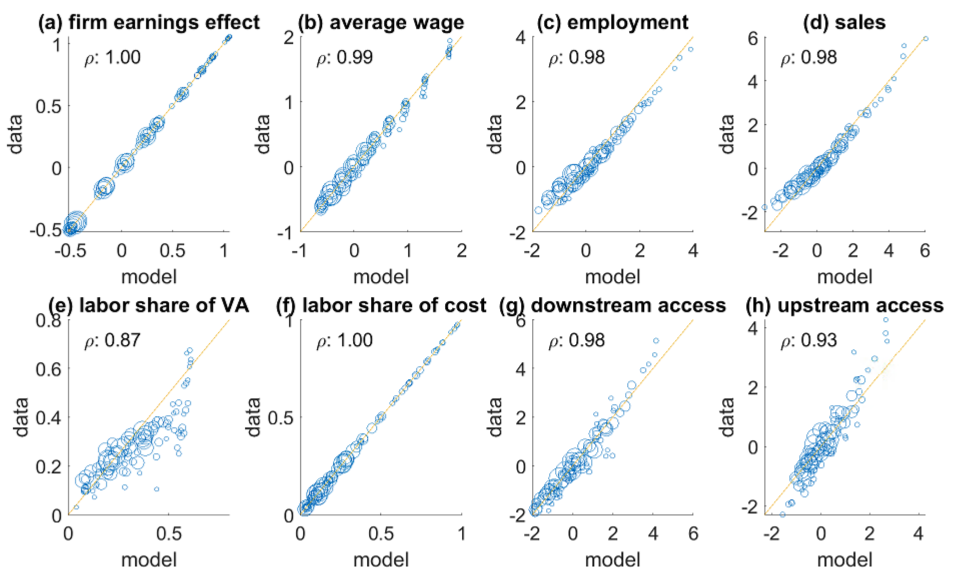
$$W_{it}^{\gamma+\epsilon}\left[\lambda\left(W_{it}/\omega_{it}\right)^{1-\epsilon} + (1-\lambda)\, S_{it}^{\epsilon-1}\right]^{\frac{\sigma-\epsilon}{1-\epsilon}} \bar{\phi}_{it} = \frac{\lambda}{\mu^{\sigma}\eta^{\gamma}} D_{it} T_{it}^{\sigma-1}\omega_{it}^{\epsilon-1} \tag{F.1}$$

which has a unique solution for W_{it} given $\left\{D_{it}, S_{it}, \bar{\phi}_{it}\right\}$. Solutions for ν_{it} and X_{it} are then easy to recover given W_{it}.

G Model Fit

Figure 5 shows the fit of the model's baseline equilibrium to key moments in the data, where each circle in a plot represents a firm group and the size of each circle is increasing in the number of firms in the group. The figure also shows the correlation (ρ) between each variable in the data and model at the firm group level, weighted by the number of firms in each group.

Figure 5: Fit of the baseline equilibrium to key empirical moments



Notes: Each marker in the figure represents a firm group, with the size of each marker increasing in the number of firms in the group. ρ indicates the correlation between model and data moments at the firm group level, weighted by the number of firms in each group.