Online Appendix for "Efficiency and Incidence of Taxation with Free Entry and Love-of-Variety Preferences"

A Specific Taxation and Ad Valorem and Results

Proof. ***Marginal Excess Burden Formula for specific tax*** $\frac{dW}{dt}$**.**

Let the total welfare to be the sum of consumer surplus, profits and government tax revenues.

$$W(p(t), t, J(t)) = \underbrace{u(Q_L(t), J(t)) - (p(t)(1+\tau)+t)Q_L(t)}_{CS} \\ + \underbrace{p(t)Q_L(t) - J(t)c\left(\frac{Q_L(t)}{J(t)}\right) - J(t)F}_{J\pi} + \underbrace{tQ_L(t) + p(t)\tau Q}_{R}$$

By totally differentiating $W_L(t) = W(p(t), t, J(t))$ with respect to t (and keeping τ constant) we obtain

$$\frac{dW_L}{dt} = \left(\frac{\partial u}{\partial Q}(Q_0, J_0) - c'(q_0)\right)\frac{dQ_L}{dt} + \left(\frac{\partial u}{\partial J}(Q_0, J_0) - c(q_0) - F + q_0 c'(q_0)\right)\frac{dJ}{dt} \\ = (p_0(1+\theta_\tau \tau_0) + \theta_t t_0 - c'(q_0))\frac{dQ_L}{dt} + (\Lambda_0 + \pi_0 - [p_0 - c'(q_0)] * q_0)\frac{dJ}{dt} \tag{1}$$

where we used the first-order approximation from Chetty, Looney and Kroft (2009) $\frac{\partial u}{\partial J}(Q_0, J_0) = p_0(1+\theta_\tau\tau_0) + \theta_t t_0$, we used our definition of variety effect $\Lambda_0 = \frac{\partial u}{\partial J}(Q_0, J_0)$ and profits $\pi_0 = p_0 q_0 - c(q_0) - F$. When $t_0 = 0$, $p_0 = c'(q^*)$ and $\Lambda_0 = -\pi_0$, we get $\frac{dW_L}{dt} = 0$ which is the first-best outcome. □

Proof. ***Lemma 1.***

Let $\pi = pq - c(q) = 0$ be the free-entry condition of firms. When τ is constant, then $\frac{d\pi}{dt} = 0$ implies that$(p - mc)\frac{dq}{dt} = -q\frac{dp}{dt}$ and so $\frac{p-mc}{p} = -\frac{q/t}{p/t}\frac{\frac{dp}{dt}}{\frac{dq}{dt}}$.

If t is now constant, then $\frac{d\pi}{d\tau} = 0$ implies $(p - mc)\frac{dq}{d\tau} = -q\frac{dp}{d\tau}$ and so $\frac{p-mc}{p} = -\frac{q/\tau}{p/\tau}\frac{\frac{dp}{d\tau}}{\frac{dq}{d\tau}}$. □

Proof. ***Proposition 1.***

Let $\Delta = \left[2 - \frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right] - \frac{\epsilon_D \Lambda}{(p(1+\tau)+t)q}\left(1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \frac{1}{\epsilon_{ms}}\right) + \left(1 - \frac{\nu_q}{J}\right)\epsilon_D \frac{JQ}{p(1+\tau)+t}\frac{\partial^2 P}{\partial J \partial Q}$.[1] The firm stability conditions $\frac{\partial^2 \pi_j}{\partial p_j^2} < 0$ and $\frac{\partial \pi_j}{\partial J} < 0$, are respectively equivalent to $1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}} > 0$ and $\Delta > 0$, where $\epsilon_D^* = \frac{p(1+\theta_\tau \tau)}{p(1+\tau)+t}\epsilon_D$. Here, Δ and ϵ_D^* are written in the general form that depends on both the specific tax rate t and the ad valorem tax rate τ.

By Lemma 1, we have $\frac{dPS}{dt} = 0$. Therefore substituting this into equation (1) we obtain:

$$\frac{dW}{dt} = \Lambda_0 \frac{dJ}{dt} - Q_0 \frac{dp}{dt} + (\theta_t t_0 + p_0 \theta_\tau \tau_0)\frac{dQ_L}{dt}$$

From the behavioral equation of consumers $wtp(Q, J) \equiv P(Q, J) = p(1 + \theta_\tau \tau) + \theta_t t$, we have

$$mwtp(Q, J)\frac{dQ}{dt} + \frac{\partial P}{\partial J}\frac{dJ}{dt} = \frac{dp}{dt}(1 + \theta_\tau \tau) + \theta_t \tag{2}$$

In addition, from the free-entry condition, $(p - mc)\frac{dq}{dt} = -q\frac{dp}{dt}$, and firm's first-order condition, $p - mc = -mwtp(Q, J)Q\frac{\nu_q}{J(1+\theta_\tau \tau)}$, we have

$$mwtp(Q, J)\nu_q \frac{dq}{dt} = (1 + \theta_\tau \tau)\frac{dp}{dt} \tag{3}$$

Combining this with the behavioral equation above, and letting $mwtp(Q, J) = mwtp(Q)$ for simplicity, we have

$$\begin{aligned} mwtp(Q)\nu_q \frac{dq}{dt} &= mwtp(Q)\frac{dQ}{dt} + \frac{\partial P}{\partial J}\frac{dJ}{dt} - \theta_t \\ &= mwtp(Q)\left(J\frac{dq}{dt} + q\frac{dJ}{dt}\right) + \frac{\partial P}{\partial J}\frac{dJ}{dt} - \theta_t \end{aligned} \tag{4}$$

where the second line follows from substituting $\frac{dQ}{dt} = J\frac{dq}{dt} + q\frac{dJ}{dt}$. Therefore,

$$\frac{dq}{dt} = \frac{\theta_t - \left(\frac{\partial P}{\partial J} + q * mwtp(Q)\right)\frac{dJ}{dt}}{mwtp(Q)(J - \nu_q)} \tag{5}$$

[1] This becomes $\Delta = \left[2 - \frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right] - \frac{\Lambda \epsilon_D}{(p(1+\tau)+t)q}\left(1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \frac{1}{\epsilon_{ms}}\right)$ under parallel demands.

Using now $\frac{dq}{dt} = \frac{\partial q}{\partial t} + \frac{\partial q}{\partial J}\frac{dJ}{dt}$ (note that $\frac{\partial q}{\partial t} = \left.\frac{dq}{dt}\right|_J$) we can get

$$\frac{dJ}{dt} = \frac{\theta_t - (J - \nu_q)mwtp(Q)\frac{\partial q}{\partial t}}{\frac{\partial P}{\partial J} + q * mwtp(Q) + (J - \nu_q)mwtp(Q)\frac{\partial q}{\partial J}} \tag{6}$$

From Kroft et al. (2020), we have

$$\frac{\partial q}{\partial t} = \left.\frac{dq}{dt}\right|_J = \frac{1}{Jmwtp(Q)}\left(\rho_t^{SR} + \theta_t - 1\right) = \frac{\omega_t^{SR}\theta_t}{Jmwtp(Q)} \tag{7}$$

where $\rho_t^{SR} = 1 - (1 - \omega_{SR})\,\theta_t$ and $\omega_{SR} = \frac{1}{1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}}$, where $\epsilon_D^* = \frac{p(1+\theta_\tau \tau)}{p(1+\tau)+t}\epsilon_D$ (short-run passthrough is taken from Kroft et al. (2020)).

Finally, fix t, and differentiate the first-order condition $(p - mc)(1+\theta_\tau \tau) + mwtp(Q,J)Q\frac{\nu_q}{J} = wtp(Q,J) - \theta_t t - mc(1+\theta_\tau \tau) + mwtp(Q,J)Q\frac{\nu_q}{J} = 0$ with respect to J to get:

$$\frac{\partial P}{\partial J} + mwtp(Q)\left(q + J\frac{\partial q}{\partial J}\right) - c''(q)(1+\theta_\tau \tau)\frac{\partial q}{\partial J} + \frac{\partial q}{\partial J}mwtp(Q)\nu_q + q\nu_q mwtp'(Q)\left(q + J\frac{\partial q}{\partial J}\right) + \frac{\partial^2 P}{\partial J \partial Q}q\nu_q = 0$$

where we have assumed that $\frac{\partial \nu}{\partial J} = 0$. Further simplifying yields:

$$\frac{\partial q}{\partial J} = -\frac{\frac{\partial P}{\partial J} + \frac{\partial^2 P}{\partial J \partial Q}q\nu_q + mwtp(Q)q + q^2\nu_q mwtp'(Q)}{(J + \nu_q)mwtp(Q) - c''(q)(1+\theta_\tau \tau) + Jq\nu_q mwtp'(Q)} \tag{8}$$

Rearranging equation (8), the denominator is equal to $J * mwtp(Q) * \left(1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)$, and so we get:

$$\frac{\partial q}{\partial J} = -\frac{\omega_{SR}}{J * mwtp(Q)}\left(\frac{\partial P}{\partial J} + \frac{\nu_q}{J}Q\frac{\partial^2 P}{\partial J \partial Q}\right) - \frac{q}{J}\omega_{SR}\left(1 - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right) \tag{9}$$

Note:

$$\begin{aligned}\omega_{SR} * \frac{\nu_q}{J} * q * mwtp(Q) * \Delta = {}& \frac{\partial P}{\partial J}\left(1 - \omega_{SR}\left(1 - \frac{\nu_q}{J}\right)\right) - \omega_{SR}\frac{\nu_q}{J}\left(1 - \frac{\nu_q}{J}\right)\frac{\partial^2 P}{\partial J \partial Q}Q \\ & + q * mwtp(Q)\left(1 - \omega_{SR}\left(1 - \frac{\nu_q}{J}\right)\left(1 - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)\right)\end{aligned}$$

Substituting equation (9) and equation (7) into equation (6), we get:

$$\frac{dJ}{dt} = \theta_t \left(\frac{1 - \omega_{SR}\left(1 - \frac{\nu_q}{J}\right)}{\omega_{SR}\frac{\nu_q}{J}\Delta} \right)$$

and substituting $\frac{dJ}{dt}$ into equation (5), we obtain:

$$\frac{dq}{dt} = \frac{\theta_t}{J * mwtp(Q)} \left(\frac{1 - \frac{1}{\epsilon_{ms}}}{\Delta} \right)$$

Finally, from equation (3) and the expression for $\frac{dq}{dt}$ we have:

$$\begin{aligned} \rho_t &= 1 + mwtp(Q, J)\nu_q \frac{dq}{dt} \\ &= \frac{\Delta + \frac{\nu_q}{J}\theta_t\left(1 - \frac{1}{\epsilon_{ms}}\right)}{\Delta} \end{aligned}$$

□

Proof. ***Corollary 1.***

The proof is immediate by setting $\theta_t = \theta_\tau = 1$, $\Lambda_0 = 0$ and $t_0 = \tau_0 = 0$ into the conditions of Proposition 1. □

Proof. ***Proposition 2***

Consider a change in the tax from τ_0 to τ_1. A first-order approximation to the marginal excess burden of taxation is:

$$\frac{dW}{d\tau} = \underbrace{(p_0(1 + \theta_\tau\tau_0) + \theta_t t_0 - c'(q_0))\frac{dQ_L}{d\tau}}_{\text{Quantity effect}} + \underbrace{(\Lambda_0 + \pi_0 - [p_0 - c'(q_0)] * q_0)\frac{dJ}{d\tau}}_{\text{Diversity effect}} \tag{10}$$

Under Lemma 1, the marginal excess burden of taxation is given by:

$$\frac{dW}{d\tau} = \Lambda_0\frac{dJ}{d\tau} - Q_0\frac{dp}{d\tau} + (\theta_t t_0 + p_0\theta_\tau\tau_0)\frac{dQ_L}{d\tau} \tag{11}$$

Willingness-to-pay with ad valorem taxes takes the form $wtp(Q) = p(1+\theta_\tau \tau)$, so $mwtp(Q)\frac{dQ}{d\tau} + \frac{\partial P}{\partial J}\frac{dJ}{d\tau} = \frac{dp}{d\tau}(1+\theta_\tau\tau) + p\theta_\tau$. We have the free entry-condition $(p - mc)\frac{dq}{d\tau} = -q\frac{dp}{d\tau}$, and the firm's first-order condition $p - mc = -\frac{\nu_q}{J(1+\theta_\tau\tau)} mwtp(Q)Q$. Therefore, we have:

$$\nu_q * mwtp(Q)\frac{dq}{d\tau} = (1+\theta_\tau\tau)\frac{dp}{d\tau} \tag{12}$$

which implies:

$$\frac{dq}{d\tau} = \frac{p\theta_\tau - \left(\frac{\partial P}{\partial J} + q * mwtp(Q)\right)\frac{dJ}{d\tau}}{mwtp(Q)\left(1 - \frac{\nu_q}{J}\right)} \tag{13}$$

Using now $\frac{dq}{d\tau} = \frac{\partial q}{\partial \tau} + \frac{\partial q}{\partial J}\frac{dJ}{d\tau}$ (Here $\frac{\partial q}{\partial \tau} = \frac{dq}{d\tau}\Big|_J$). we get

$$\frac{dJ}{d\tau} = \frac{p\theta_\tau + \Big(\nu_q - J\Big)mwtp(Q)\frac{\partial q}{\partial \tau}}{\frac{\partial P}{\partial J} + q * mwtp(Q) + \Big(J - \nu_q\Big)\frac{\partial q}{\partial J}} \tag{14}$$

We also have

$$\frac{\partial q}{\partial \tau} = \frac{dq}{d\tau}\bigg|_J = \frac{1}{Jmwtp(Q)}\left(\theta_\tau mc * \omega_{SR}\right)$$

where $\rho_\tau^{SR} = 1 - \Big(1 - \omega_{SR}\frac{mc}{p}\Big)\theta_\tau$ and $\omega_{SR} = \frac{1}{1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}}$. Moreover,

$$\frac{\partial q}{\partial J} = -\frac{\omega_{SR}}{J * mwtp(Q)}\left(\frac{\partial P}{\partial J} + \frac{\partial^2 P}{\partial J \partial Q} q\nu_q\right) - \frac{q\omega_{SR}}{J}\left(1 - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right) \tag{15}$$

Therefore, substituting $\frac{\partial q}{\partial \tau}$ and $\frac{\partial q}{\partial J}$ into equation (14)we have

$$\begin{aligned}
\frac{dJ}{d\tau} &= \theta_\tau\left(\frac{p - mc * \omega_{SR}\Big(1 - \frac{\nu_q}{J}\Big)}{\omega_{SR} * \frac{\nu_q}{J} * q * mwtp(Q) * \Delta}\right) \\
&= p\theta_\tau\left(\frac{\left(1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right) - \left(1 - \frac{\frac{\nu_q}{J}}{\epsilon_D^*}\right)\left(1 - \frac{\nu_q}{J}\right)}{\frac{\nu_q}{J} * q * mwtp(Q) * \Delta}\right) \\
&= -\frac{p\theta_\tau J\epsilon_D}{p(1+\tau)+t}\left(\frac{1 + \frac{1}{\epsilon_D^*} - \frac{\frac{\nu_q}{J}}{\epsilon_D^*} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S\frac{\nu_q}{J}} + \frac{1}{\epsilon_{ms}}}{\Delta}\right)
\end{aligned} \tag{16}$$

Recall $\Delta = \left[2 - \frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right] - \frac{\epsilon_D J \frac{\partial P}{\partial J}}{(p(1+\tau)+t)} \left(1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \frac{1}{\epsilon_{ms}}\right) + \left(1 - \frac{\nu_q}{J}\right) \epsilon_D \frac{JQ}{p(1+\tau)+t} \frac{\partial^2 P}{\partial J \partial Q}$.
Substituting equation (16) into equation (13), then:

$$\begin{aligned}
\frac{dq}{d\tau} &= \frac{-\theta_\tau \omega_{SR}}{Jmwtp(Q)} \left(\frac{\frac{\partial P}{\partial J}(p - mc) + q * mwtp(Q)\left(p\left(1 - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right) - mc\right)}{\omega_{SR}\frac{\nu_q}{J}\Delta} \right) \\
&= \frac{-p\theta_\tau}{Jmwtp(Q)} \left(\frac{\frac{\frac{\nu_q}{J}}{\epsilon_D^*} - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}} - \frac{J\frac{\partial P}{\partial J}\epsilon_D}{((1+\tau)p+t)}\frac{\frac{\nu_q}{J}}{\epsilon_D^*}}{\frac{\nu_q}{J}\Delta} \right)
\end{aligned}$$

Finally,

$$\begin{aligned}
\rho_\tau &= \frac{1}{p}\frac{1+\tau}{1+\theta_\tau\tau}\nu_q mwtp(Q)\frac{dq}{d\tau} + 1 \\
&= -\frac{\nu_q}{J}\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)} \left(\frac{\frac{\partial P}{\partial J}\left(\frac{p-mc}{p}\right) + q * mwtp(Q)\left(\frac{p-mc}{p} - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)}{\frac{\nu_q}{J}\Delta} \right) + 1 \\
&= \frac{\frac{\nu_q}{J}\Delta - \frac{\nu_q}{J}\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)}\left(\frac{p-mc}{p} - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right) + \frac{J\frac{\partial P}{\partial J}\epsilon_D}{((1+\tau)p+t)}\left(\frac{\nu_q}{J}\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)}\frac{p-mc}{p}\right)}{\frac{\nu_q}{J}\Delta}
\end{aligned}$$

Using $\frac{p-mc}{p} = \frac{\frac{\nu_q}{J}}{\epsilon_D^*}$, we obtain:

$$\rho_\tau = \frac{\Delta + \frac{\nu_q}{J}\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)}\left[1 - \frac{1}{\epsilon_D^*} - \frac{1}{\epsilon_{ms}} + \frac{J\frac{\partial P}{\partial J}}{(1+\tau)p+t}\left(\frac{\epsilon_D}{\epsilon_D^*}\right)\right]}{\Delta}$$

□

Derivation of Δ

Let $\Delta = \left[2 - \frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right] - \left(1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \frac{1}{\epsilon_{ms}}\right) \epsilon_D \frac{J}{p(1+\tau)+t} \frac{\partial P}{\partial J} + \left(1 - \frac{\nu_q}{J}\right) \epsilon_D \frac{JQ}{p(1+\tau)+t} \frac{\partial^2 P}{\partial J \partial Q}$, we show that $\Delta = -\frac{J\epsilon_D^*}{pq} \frac{\partial \pi}{\partial J}$.

Proof. The effect of taxes on entry is derived by using the implicit function theorem on the long-run entry condition $\pi(q(J,t,\tau_0), J, t, \tau_0) = 0$, and the first-order condition of the firm $\frac{\partial \pi}{\partial q} = 0$, so that $\frac{dJ}{d\tau} = -\frac{\frac{\partial \pi}{\partial \tau}}{\frac{\partial \pi}{\partial J}}$. Therefore

$$
\begin{aligned}
\frac{\partial \pi}{\partial J} &= -\frac{\frac{\partial \pi}{\partial t}}{\frac{dJ}{dt}} \\
&= \frac{\frac{p\theta_\tau q - mwtp(Q)*Q*\left(1-\frac{\nu_q}{J}\right)\frac{\partial q}{\partial \tau}}{1+\theta_\tau \tau}}{\frac{p\theta_\tau - mwtp(Q)*J*\left(1-\frac{\nu_q}{J}\right)\frac{\partial q}{\partial \tau}}{\frac{\partial P}{\partial J} + q*mwtp(Q) + mwtp(Q)*J*\left(1-\frac{\nu_q}{J}\right)\frac{\partial q}{\partial J}}} \\
&= \frac{q}{1+\theta_\tau \tau}\left(\frac{\partial P}{\partial J} + q*mwtp(Q) + mwtp(Q)*J*\left(1-\frac{\nu_q}{J}\right)\frac{\partial q}{\partial J}\right) \\
&= \frac{q}{1+\theta_\tau \tau} mwtp(Q) Q \frac{1}{J}(\Delta) \\
&= \frac{q(p(1+\tau)+t)}{1+\theta_\tau \tau} \frac{mwtp(Q)Q}{p(1+\tau)+t} \frac{1}{J}(\Delta) \\
&= -\frac{1}{\epsilon_D^*} \frac{pq}{J}(\Delta)
\end{aligned}
$$

□

Corollary. ***A1.*** *Consider the case of full-optimization ($\theta_\tau = \theta_t = 1$), homogeneous products ($\Lambda_0 = 0$) and no pre-existing taxes ($\tau_0 = t_0 = 0$). The marginal excess burden and pass-*

through formulas are given respectively by:

$$\frac{dW}{d\tau} = -Q_0 \frac{dp}{d\tau} \tag{17}$$

$$\rho_\tau = \frac{2 + \frac{\epsilon_D^* - \frac{\nu_q}{J_0}}{\epsilon_S \frac{\nu_q}{J_0}} - \frac{\frac{\nu_q}{J_0}}{\epsilon_D^*}}{2 - \frac{\nu_q}{J_0} + \frac{\epsilon_D^* - \frac{\nu_q}{J_0}}{\epsilon_S \frac{\nu_q}{J_0}} + \frac{\frac{\nu_q}{J_0}}{\epsilon_{ms}}} \tag{18}$$

$$\frac{1}{p_0}\frac{dJ}{d\tau} = -\frac{J\epsilon_D}{p_0}\left[\frac{1 + \frac{\epsilon_D^* - \frac{\nu_q}{J_0}}{\epsilon_S \frac{\nu_q}{J_0}} + \frac{1}{\epsilon_{ms}} + \frac{1 - \frac{\nu_q}{J_0}}{\epsilon_D^*}}{2 - \frac{\nu_q}{J_0} + \frac{\epsilon_D^* - \frac{\nu_q}{J_0}}{\epsilon_S \frac{\nu_q}{J_0}} + \frac{\frac{\nu_q}{J_0}}{\epsilon_{ms}}}\right] \tag{19}$$

Proof. ***Corollary A1.***

The proof is immediate by setting $\theta_\tau = \theta_t = 1$, $\Lambda_0 = 0$ and $\tau_0 = t_0 = 0$ into the conditions of Proposition 2. □

Lemma. ***A1.*** *For fixed* τ*. The effect of competition on prices and output is given respectively by:*

$$\frac{\partial p}{\partial J} = \left[\frac{\partial P}{\partial J} - \frac{p+t}{J\epsilon_D}\left(1 + \frac{J}{q}\frac{\partial q}{\partial J}\right)\right]$$

$$\frac{J}{q}\frac{\partial q}{\partial J} = -\omega_{SR}\left[1 - \frac{\nu_q}{J}\left(1 - \frac{1}{\epsilon_{ms}}\right) - \frac{J\epsilon_D}{(1+\tau)p + tq}\frac{\partial P}{\partial J}\right]$$

Thus, in the case of constant marginal cost ($\epsilon_S = \infty$), $\frac{\partial p}{\partial J} < 0$ *if and only if* $\frac{1}{\epsilon_{ms}} \frac{\Lambda\epsilon_D}{(p+t)q} < 1$ *and there is business stealing* $\left(\frac{\partial q}{\partial J} < 0\right)$ *whenever* $\frac{\Lambda\epsilon_D}{(p+t)q} + \frac{\nu_q}{J}\left(1 - \frac{1}{\epsilon_{ms}}\right) < 1$.

For fixed t. The effect of competition on prices and output is given respectively by:

$$\frac{\partial p}{\partial J} = \frac{1}{1+\theta_\tau\tau}\left[\frac{\partial P}{\partial J} - \frac{p(1+\tau)}{J\epsilon_D}\left(1 + \frac{J}{q}\frac{\partial q}{\partial J}\right)\right]$$

$$\frac{J}{q}\frac{\partial q}{\partial J} = -\omega_{SR}\left[1 - \frac{\nu_q}{J}\left(1 - \frac{1}{\epsilon_{ms}}\right) - \frac{J\epsilon_D}{(1+\tau)p + t}\frac{\partial P}{\partial J}\right]$$

Thus, in the case of constant marginal cost ($\epsilon_S = \infty$), $\frac{\partial p}{\partial J} < 0$ *if and only if* $\left(\frac{1}{\epsilon_{ms}}\right)\frac{\epsilon_D}{p(1+\tau)} J \frac{\partial P}{\partial J} < 1$ *and there is business stealing* $\left(\frac{\partial q}{\partial J} < 0\right)$ *whenever* $\frac{\epsilon_D}{(1+\tau)p} J \frac{\partial P}{\partial J} + \frac{\nu_q}{J}\left(1 - \frac{1}{\epsilon_{ms}}\right) < 1$*. Furthermore, assuming parallel demands* $\frac{\partial P}{\partial J} = \frac{\Lambda}{Q}$.

Proof. ***Lemma A1. Unit Taxes:***

From the behavioral equation $wtp(Q) = P(Q, J) = p + \theta t$, we can express price as a function of J and t. Then we have

$$p(J, t) = P(Q(J, t), J) - \theta t$$

Therefore,

$$\begin{aligned}\frac{\partial p}{\partial J} &= \frac{\partial P}{\partial J} + mwtp(Q, J)\frac{\partial Q}{\partial J} \\ &= \frac{\partial P}{\partial J} + q * mwtp(Q, J) + mwtp(Q, J) * J * \frac{\partial q}{\partial J} \\ &= \left[\frac{\partial P}{\partial J} - \frac{p+t}{J\epsilon_D}\left(1 + \frac{J}{q}\frac{\partial q}{\partial J}\right)\right]\end{aligned}$$

From the proof of Proposition 1, we also have that:

$$\begin{aligned}\frac{\partial q}{\partial J} &= -\frac{\frac{\Lambda}{Q} + mwtp(Q)q + q^2\nu_q mwtp'(Q)}{(J + \nu_q)mwtp(Q) - c''(q) + Jq\nu_q mwtp'(Q)} \\ &= -\frac{\omega_{SR}\Lambda}{JQ * mwtp(Q)} - \frac{q}{J}\omega_{SR}\left(1 - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{\partial p}{\partial J} &= \left[\frac{\Lambda}{Q} - \frac{p+t}{J\epsilon_D}\left(1 + \frac{J}{q}\frac{\partial q}{\partial J}\right)\right] \\ \frac{J}{q}\frac{\partial q}{\partial J} &= -\omega_{SR}\left[1 - \frac{\nu_q}{J}\left(1 - \frac{1}{\epsilon_{ms}}\right) - \frac{\Lambda\epsilon_D}{(p+t)q}\right]\end{aligned}$$

Ad valorem:

The proof is analogous to Lemma 2. The only modification is that the behavioral equation for ad valorem taxation $p(J, t) = \frac{P(Q(J,t),J)}{1+\theta_\tau \tau}$ implies a rescaling is needed for $\frac{\partial p}{\partial J}$. □

B Comparison between Ad Valorem and Specific Taxation

We begin by considering the reduced-form effects of taxes in order to compare ad valorem to specific taxation. Throughout we will make use of the definitions $\epsilon_D = -\frac{p(1+\tau)+t}{Qmwtp(Q)}$, $\epsilon_D^* = \frac{p(1+\theta_\tau \tau)}{p(1+\tau)+t}\epsilon_D$, and $\Delta = \left[2 - \frac{\nu_q}{J} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right] - \left(1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \frac{1}{\epsilon_{ms}}\right)\epsilon_D \frac{J}{(p(1+\tau)+t)q}\frac{\partial P}{\partial J} + \left(1 - \frac{\nu_q}{J}\right)\epsilon_D \frac{JQ}{p(1+\tau)+t}\frac{\partial^2 P}{\partial J \partial Q} > 0$ for the stability condition:

$$
\begin{aligned}
\rho_t &= \frac{\Delta + \theta_t \frac{\nu_q}{J}\left(1 - \frac{1}{\epsilon_{ms}}\right)}{\Delta} \\
\rho_\tau &= \frac{\Delta + \frac{\nu_q}{J}\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau \tau)}\left(1 - \frac{1}{\epsilon_{ms}} + \frac{1}{\epsilon_D^*}\left(\frac{J\epsilon_D}{(p(1+\tau)+t)}\frac{\partial P}{\partial J} - 1\right)\right)}{\Delta} \\
\frac{dq}{dt} &= -\frac{\theta_t q \epsilon_D}{p(1+\tau)+t}\left(\frac{1 - \frac{1}{\epsilon_{ms}}}{\Delta}\right) \\
\frac{dq}{d\tau} &= -\frac{\theta_\tau p q \epsilon_D}{p(1+\tau)+t}\left(\frac{1 - \frac{1}{\epsilon_{ms}} - \frac{1}{\epsilon_D^*} + \frac{J\epsilon_D}{(p(1+\tau)+t)}\frac{\partial P}{\partial J}\frac{1}{\epsilon_D^*}}{\Delta}\right) \\
\frac{dJ}{dt} &= -\frac{\theta_t J \epsilon_D}{p(1+\tau)+t}\left(\frac{1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \frac{1}{\epsilon_{ms}}}{\Delta}\right) \\
\frac{dJ}{d\tau} &= -\frac{\theta_\tau p J \epsilon_D}{p(1+\tau)+t}\left(\frac{1 + \frac{1}{\epsilon_D^*} - \frac{\frac{\nu_q}{J}}{\epsilon_D^*} + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \frac{1}{\epsilon_{ms}}}{\Delta}\right) \\
\frac{dQ}{dt} &= -\frac{\theta_t Q \epsilon_D}{p(1+\tau)+t}\left(\frac{2 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}}}{\Delta}\right) \\
\frac{dQ}{d\tau} &= -\frac{\theta_\tau p Q \epsilon_D}{p(1+\tau)+t}\left(\frac{2 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \left(\frac{J\epsilon_D}{(p(1+\tau)+t)}\frac{\partial P}{\partial J} - \frac{\nu_q}{J}\right)\frac{1}{\epsilon_D^*}}{\Delta}\right) \\
\frac{dW}{dt} &= \Lambda \frac{dJ}{dt} + \theta_t t \frac{dQ}{dt} - Q\frac{dp}{dt} \\
\frac{dW}{d\tau} &= \Lambda \frac{dJ}{d\tau} + \theta_\tau \tau p \frac{dQ}{d\tau} - Q\frac{dp}{d\tau} \\
\frac{dR}{dt} &= Q + t\frac{dQ}{dt} \\
\frac{dR}{d\tau} &= pQ + \tau p \frac{dQ}{d\tau} + \tau Q \frac{dp}{d\tau}
\end{aligned}
$$

Proof. ***Proposition 3.*** Rewrite ρ_τ as:

$$\rho_\tau = \frac{\left[2 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} - \left(1 - \frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)}\right)\left(\frac{\nu_q}{J} - \frac{\frac{\nu_q}{J}}{\epsilon_{ms}}\right)\right]}{\Delta} - \frac{\frac{J\epsilon_D}{p(1+\tau)+t}\frac{\partial P}{\partial J}\left(1 + \frac{\epsilon_D^* - \frac{\nu_q}{J}}{\epsilon_S \frac{\nu_q}{J}} + \frac{1}{\epsilon_{ms}}\right) + \frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)}\frac{\frac{\nu_q}{J}}{\epsilon_D^*}\left[\frac{J\epsilon_D}{p(1+\tau)+t}\frac{\partial P}{\partial J} - 1\right]}{\Delta}$$

Then, observe that for $\theta_t = \frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)}$ (for example if $\theta_t = \theta_\tau$ and $\tau = 0$) then

$$\rho_\tau - \rho_t = \frac{\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau\tau)}\frac{\frac{\nu_q}{J}}{\epsilon_D^*}\left[\frac{J\epsilon_D}{p(1+\tau)+t}\frac{\partial P}{\partial J} - 1\right]}{\Delta}$$

so

$$\rho_\tau > \rho_t \Leftrightarrow \frac{J\epsilon_D}{p(1+\tau)+t}\frac{\partial P}{\partial J} > 1 \Leftrightarrow \frac{\Lambda}{Q} + q * mwtp(Q) > 0$$

We now consider the marginal cost of public funds (MCPF) starting from zero initial taxes.

$$R = \tau p Q + t Q$$

$$\begin{aligned} MCPF_t &= -\frac{\Lambda\frac{dJ}{dt} + \theta_t t\frac{dQ}{dt} - Q\frac{dp}{dt}}{Q + t\frac{dQ}{dt}} \\ &= -\frac{\Lambda}{Q}\frac{dJ}{dt} + \frac{dp}{dt} \\ &= -\frac{\Lambda}{Q}\frac{dJ}{dt} + \rho_t - 1 \end{aligned}$$

$$\begin{aligned} MCPF_\tau &= -\frac{\Lambda\frac{dJ}{d\tau} + \theta_\tau \tau p\frac{dQ}{d\tau} - Q\frac{dp}{d\tau}}{pQ + \tau p\frac{dQ}{d\tau} + \tau Q\frac{dp}{d\tau}} \\ &= -\frac{\Lambda}{pQ}\frac{dJ}{d\tau} + \rho_\tau - 1 \end{aligned}$$

Furthermore,

$$\frac{dJ}{dt} = \frac{\theta_t}{\frac{\partial P}{\partial J} + q * mwtp(Q)} + \frac{1 - \frac{1}{\frac{\nu_q}{J}}}{\frac{\partial P}{\partial J} + q * mwtp(Q)} \frac{dp}{dt}$$

$$\frac{dJ}{d\tau} = \frac{p\theta_\tau}{\frac{\partial P}{\partial J} + q * mwtp(Q)} + (1 + \theta_\tau \tau) \frac{1 - \frac{1}{\frac{\nu_q}{J}}}{\frac{\partial P}{\partial J} + q * mwtp(Q)} \frac{dp}{d\tau}$$

and when taxes are zero, we get:

$$\frac{dJ}{dt} = \frac{\theta_t}{\frac{\partial P}{\partial J} + q * mwtp(Q)} + \frac{1 - \frac{1}{\frac{\nu_q}{J}}}{\frac{\partial P}{\partial J} + q * mwtp(Q)} (\rho_t - 1)$$

$$\frac{dJ}{d\tau} = \frac{p\theta_\tau}{\frac{\partial P}{\partial J} + q * mwtp(Q)} + \frac{1 - \frac{1}{\frac{\nu_q}{J}}}{\frac{\partial P}{\partial J} + q * mwtp(Q)} p(\rho_\tau - 1)$$

and so

$$MCPF_t = -\frac{\Lambda}{Q} \frac{\theta_t}{\frac{\partial P}{\partial J} + q * mwtp(Q)} + (\rho_t - 1) \left(1 - \frac{\Lambda}{Q} \frac{1 - \frac{1}{\frac{\nu_q}{J}}}{\frac{\partial P}{\partial J} + q * mwtp(Q)} \right)$$
$$MCPF_\tau = -\frac{\Lambda}{Q} \frac{\theta_\tau}{\frac{\partial P}{\partial J} + q * mwtp(Q)} + (\rho_\tau - 1) \left(1 - \frac{\Lambda}{Q} \frac{1 - \frac{1}{\frac{\nu_q}{J}}}{\frac{\partial P}{\partial J} + q * mwtp(Q)} \right)$$

Assuming $\theta_t = \theta_\tau$ and $\tau = t = 0$, and $\frac{\partial P}{\partial J} = \frac{\Lambda}{Q}$, note that $1 - \frac{\Lambda}{Q} \frac{1 - \frac{1}{\frac{\nu_q}{J}}}{\frac{\Lambda}{Q} + q * mwtp(Q)} = \left(\frac{q * mwtp(Q) + \frac{\frac{\Lambda}{Q}}{\frac{\nu_q}{J}}}{\frac{\Lambda}{Q} + q * mwtp(Q)} \right)$.

Therefore:

$$sign(MCPF_\tau - MCPF_t) = sign\left((\rho_\tau - \rho_t) * \frac{q * mwtp(Q) + \frac{\frac{\Lambda}{Q}}{\frac{\nu_q}{J}}}{\frac{\Lambda}{Q} + q * mwtp(Q)} \right)$$
$$= sign\left(q * mwtp(Q) + \frac{\frac{\Lambda}{Q}}{\frac{\nu_q}{J}} \right)$$

Finally, observe:

$$sign\left(\frac{1}{p}\frac{dJ}{d\tau} - \frac{dJ}{dt}\right) = sign\left((\rho_\tau - \rho_t) * \frac{1 - \frac{1}{\frac{\nu_q}{J}}}{\frac{\partial P}{\partial J} + q * mwtp(Q)}\right)$$
$$< 0$$

□

C Microfoundations for Demand

In this section, we provide the microfoundation for parallel demands. First, we introduce a class of continuous choice models that are nested by our utility function.

Preferences. Let the representative consumer's utility function given by

$$u_J(q_1, \ldots, q_J, m) = h_J(q_1, \ldots, q_J) + m$$

for any $h_J : \{1, \ldots, J\} \to \mathbb{R}$ which is symmetric in all its arguments, continuously differentiable, strictly quasi-concave and $h(0, \ldots, 0) = 0$ and where the linear good m is interpreted as money.

Demand. The consumer's problem is

$$\max u_J(q_1, \ldots, q_J, m) = h_J(q_1, \ldots, q_J) + m \tag{20}$$
$$\text{subject to } m + \sum_{j=1}^{J} p_j q_j = y.$$

When the consumer is facing symmetric prices $p_j = p$ for all j, we can transform the problem as follows. Define $H_J(Q) = h_J\left(\frac{Q}{J}, \ldots, \frac{Q}{J}\right)$ where we interpret Q as aggregate demand.

The new problem then is given by

$$u^*(p, J, y) = \max_Q H_J(Q) + y - pQ.$$

From the first-order condition, we obtain the family of inverse demands $P(Q, J) = H'_J(Q)$. Furthermore, it is easy to see that given the optimal aggregate quantity $Q(p, J)$ for price p, the strict quasi-concavity of h_J implies the consumer chooses symmetric quantities $q_j = \frac{Q}{J}$ for all j in the original problem.

Furthermore, none of the assumptions on utility are too restrictive. We show that for any family of downward sloping aggregate demands there exists a utility function $u_J : \mathbb{R}^{J+1} \to \mathbb{R}$ satisfying the conditions above that rationalize the aggregate demands. Let $P(Q, J)$ be continuously differentiable and strictly decreasing in Q. Let H be any antiderivative $\int P(Q, J)dQ$, which exists because $P(Q, J)$ is differentiable. Then, for some $\rho \in (0, 1)$, the following is a strictly quasi-concave direct utility function that rationalizes $P(Q, J)$ for integer J when all prices p_j in the market are equal:

$$u(q_1, \ldots, q_J, m) = H\left(\left(J^{\rho-1}\sum_{j=1}^{J} q_j^{\rho}\right)^{\frac{1}{\rho}}\right) + m.$$

Furthermore, we can make sense of J as a continuous variable if we permit a continuum of varieties $q : [0, J] \to \mathbb{R}$ and let

$$u_J(q, m) = H\left(\left(\int_0^J J^{\rho-1} q^{\rho}(j)dj\right)^{\frac{1}{\rho}}\right) + m.$$

We provide two examples in the following to further illustrate the idea of parallel demands and its applications.

Example 1. Bulow and Pfleiderer (1981) obtain the following three categories of inverse demands as the unique curves with the property of constant pass-through:

1. $P(Q,J) = \alpha_J - \beta_J Q^\delta$, for $\delta > 0$,
2. $P(Q,J) = \alpha_J - \beta_J log(Q)$,
3. $P(Q,J) = \alpha_J + \beta_J Q^{1/\eta}$, for $\eta < 0$, which is the constant elasticity inverse demand shifted by the intercept α_J.

An important case is when $\beta_J = \beta$ for all J, then the inverse aggregate demands are linearly separable in J and Q and they shift in parallel as J moves.[2] The fact that these are the only class of curves for which marginal costs are passed-on in a constant fraction makes them a tractable benchmark and therefore they have been popular in applied work. Fabinger and Weyl (2016) generalize Bulow and Pfleiderer (1983) and characterize a broader class of "tractable equilibrium forms" of the form $P(Q,J) = \alpha_J + \beta Q^t + \gamma Q^u$ which allow for greater modeling flexibility. Again, as long as β and γ are independent of J, then we say that the inverse demands shift in parallel.

Example 2. This example shows that our revealed-preference approach allows for rational preferences that display *hate-of-variety* ($a'(J) < 0$). Imagine there is a marginal cost of consumption cJ for each unit of some good that is consumed; that is, for each unit consumed, the agent faces a constant cost of evaluating each of J varieties before he chooses. Preferences are given by

$$U = H\left(\sum_{j=1}^{J} q_j\right) - cJ\sum_{j=1}^{J} q_j + m$$

where H is concave. The inverse demands are then $P(Q,J) = h(Q) - cJ$ with $h = H'$ decreasing, therefore aggregate demand shifts inward as the variety increases (the intercept being $h(0) - cJ$). We can interpret this as the agent displaying a strong degree of thinking aversion or attention costs. More generally, if the inverse demands are given by $P(Q,J) = a(J) - h(Q)$ then the sign of $a'(J)$ is unrestricted.

[2] For example, for the first class one possible family of utility functions, among many, that rationalize the inverse aggregate demands is given by

$$u_J(q_1,\ldots,q_J,m) = \alpha_J\left(J^{\rho-1}\sum_{i=1}^{J} q_i^\rho\right)^{\frac{1}{\rho}} - \beta_J\frac{\left(\sum_{i=1}^{J} q_i\right)^{\delta+1}}{\delta+1} + m.$$

D Formulas in Calibration

Taking logs and rescaling by $\frac{W}{pQ}$ equation (11) we obtain the following expression which we use in Section 5 of the paper:

$$\frac{dlog(W)}{dlog(1+\tau)}\frac{W}{pQ} = \tilde{\Lambda}_0 \frac{dlog(J)}{dlog(1+\tau)} - \frac{dlog(p)}{dlog(1+\tau)} + \theta_\tau \tau_0 \frac{dlog(Q_L)}{dlog(1+\tau)} \tag{21}$$

where $\tilde{\Lambda}_0 \equiv \frac{\Lambda_0}{pQ}$.

We now show the derivation equation (18) in the paper. Note that the Lerner condition $\frac{p-mc}{p(1+\tau)} = \frac{\frac{\nu_q}{J}}{(1+\theta_\tau \tau)\epsilon_D}$ and the long-run free entry condition $\frac{\frac{dlogp}{d\tau}}{\frac{dlogq}{d\tau}} = -\frac{p-mc}{p}$ we can identify

$$\frac{\nu_q}{J} = -\epsilon_D \frac{1+\theta_\tau \tau}{1+\tau} \frac{\frac{dlogp}{d\tau}}{\frac{dlogq}{d\tau}} \tag{22}$$

We have from Proposition 2, and assuming constant mc, that

$$\frac{dJ}{d\tau} = -\frac{\theta_\tau J \epsilon_D}{(1+\tau)} \left[\frac{1 + \frac{1}{\epsilon_{ms}} + \frac{1-\frac{\nu_q}{J}}{\epsilon_D^*}}{\triangle} \right]$$

and

$$\rho_\tau = \frac{\triangle - \frac{\theta_\tau(1+\tau)}{(1+\theta_\tau \tau)} \left(\frac{\frac{\nu_q}{J}}{\epsilon_D^*} - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}} \right) + \frac{\Lambda \epsilon_D}{(p(1+\tau)+t)q} \left(\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau \tau)} \frac{\frac{\nu_q}{J}}{\epsilon_D^*} \right)}{\triangle}$$

where $\triangle \equiv 1 + \left[1 + \frac{\epsilon_D^* - \frac{\nu_q}{J_0}}{\frac{\nu_q}{J}\epsilon_S}\right] \left[1 - \frac{\Lambda \epsilon_D}{(1+\tau)pq}\right] - \frac{1}{\epsilon_{ms}} \frac{\Lambda \epsilon_D}{(1+\tau)pq} - \frac{\nu_q}{J}\left[1 - \frac{1}{\epsilon_{ms}}\right]$. Then

$$\triangle = -\frac{\theta_\tau J \epsilon_D}{(1+\tau)} \left[\frac{1 + \frac{1}{\epsilon_{ms}} + \frac{1-\frac{\nu_q}{J}}{\epsilon_D^*}}{\frac{dJ}{d\tau}} \right] = \frac{-\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau \tau)} \left(\frac{\frac{\nu_q}{J}}{\epsilon_D^*} - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}} \right) + \frac{\Lambda \epsilon_D}{(p(1+\tau)+t)q} \left(\frac{\theta_\tau(1+\tau)}{(1+\theta_\tau \tau)} \frac{\frac{\nu_q}{J}}{\epsilon_D^*} \right)}{\rho_\tau - 1}$$

And so, using $\rho_\tau - 1 = (1+\tau)\frac{dlog(p)}{d\tau}$, then

$$\frac{\Lambda \epsilon_D}{pq} \left(\frac{1}{(1+\theta_\tau \tau)} \frac{\frac{\nu_q}{J}}{\epsilon_D^*} \right) = -J\epsilon_D \frac{dlog(p)}{d\tau} \left[\frac{1 + \frac{1}{\epsilon_{ms}} + \frac{1-\frac{\nu_q}{J}}{\epsilon_D^*}}{\frac{dJ}{d\tau}} \right] + \frac{1+\tau}{(1+\theta_\tau \tau)} \left(\frac{\frac{\nu_q}{J}}{\epsilon_D^*} - \frac{\nu_q}{J} + \frac{\frac{\nu_q}{J}}{\epsilon_{ms}} \right)$$

which implies

$$\frac{\Lambda}{pq} = -\frac{\epsilon_D^*}{\epsilon_D}\left(1+\theta_\tau\tau\right)\frac{\frac{\epsilon_D}{\frac{\nu_q}{J}}\frac{dlog(p)}{d\tau}}{\frac{dlog(J)}{d\tau}}\left(1+\frac{1}{\epsilon_{ms}}+\frac{1-\frac{\nu_q}{J_0}}{\epsilon_D^*}\right)+\left(1+\tau\right)\frac{\epsilon_D^*}{\epsilon_D}\left(\frac{1}{\epsilon_D^*}-1+\frac{1}{\epsilon_{ms}}\right)$$

Now, from $\frac{\epsilon_D^*}{\epsilon_D} = \frac{1+\theta_\tau\tau}{1+\tau}$ and equation (22) we get

$$\begin{aligned}
\frac{\Lambda}{p(1+\tau)q} &= -\frac{1+\theta_\tau\tau}{1+\tau}\left(\frac{1+\theta_\tau\tau}{1+\tau}\right)\frac{-\frac{1+\tau}{1+\theta_\tau\tau}\frac{dlog(q)}{d\tau}}{\frac{dlog(J)}{d\tau}}\left(1+\frac{1}{\epsilon_{ms}}+\frac{1-\frac{\nu_q}{J_0}}{\epsilon_D^*}\right)+\frac{1+\theta_\tau\tau}{1+\tau}\left(\frac{1}{\epsilon_D^*}-1+\frac{1}{\epsilon_{ms}}\right)\\
&= \left(\frac{1+\theta_\tau\tau}{1+\tau}\right)\frac{\frac{dlog(q)}{d\tau}}{\frac{dlog(J)}{d\tau}}\left(1+\frac{1}{\epsilon_{ms}}+\frac{1-\frac{\nu_q}{J_0}}{\epsilon_D^*}\right)+\left(1+\theta_\tau\tau\right)\left(\frac{1}{\epsilon_D^*}-1+\frac{1}{\epsilon_{ms}}\right)\\
&= \left(\frac{1+\theta_\tau\tau}{1+\tau}\right)\left[\frac{\frac{dlog(q)}{d\tau}}{\frac{dlog(J)}{d\tau}}\left(1+\frac{1}{\epsilon_{ms}}+\frac{1-\frac{\nu_q}{J_0}}{\epsilon_D^*}\right)+\frac{1}{\epsilon_D^*}+\frac{1}{\epsilon_{ms}}-1\right]\\
&= \left(\frac{1+\theta_\tau\tau}{1+\tau}\right)\left[\frac{1}{\epsilon_{ms}}\left(\frac{\frac{dlog(q)}{d\tau}}{\frac{dlog(J)}{d\tau}}+1\right)+\frac{\frac{dlog(q)}{d\tau}}{\frac{dlog(J)}{d\tau}}\left(1+\frac{1-\frac{\nu_q}{J_0}}{\epsilon_D^*}\right)+\frac{1}{\epsilon_D^*}-1\right]\\
&= \left(\frac{1+\theta_\tau\tau}{1+\tau}\right)\left[\frac{1}{\epsilon_{ms}}\left(\frac{\frac{dlog(Q)}{d\tau}}{\frac{dlog(J)}{d\tau}}\right)+\frac{\frac{dlog(Q)}{d\tau}-\frac{dlog(J)}{d\tau}}{\frac{dlog(J)}{d\tau}}\left(1+\frac{1-\frac{\nu_q}{J}}{\epsilon_D^*}\right)+\frac{1}{\epsilon_D^*}-1\right]\\
&= \left(\frac{1+\theta_\tau\tau}{1+\tau}\right)\left[\frac{1}{\epsilon_{ms}}\left(\frac{\hat{\beta^Q}}{\hat{\beta^J}}\right)+\frac{\hat{\beta^Q}}{\hat{\beta^J}}\left(1+\frac{1-\frac{\nu_q}{J}}{\epsilon_D^*}\right)+\frac{\frac{\nu_q}{J}}{\epsilon_D^*}-2\right]
\end{aligned}$$

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Online Appendix Table OA.1:
Effect of Food and Nonfood Sales Taxes **[Placebo Test]**

	(1)	(2)	(3)	(4)
Panel A: Dependent variable is log Prices				
Own tax rate differential	0.187		0.166	0.045
	(0.021)		(0.018)	(0.011)
Other tax rate differential		0.150	0.120	
		(0.021)	(0.018)	
Panel B: Dependent variable is log Quantity				
Own tax rate differential	-0.844		-0.850	-0.878
	(0.258)		(0.227)	(0.173)
Other tax rate differential		-0.125	0.029	
		(0.257)	(0.227)	
Panel C: Dependent variable is log Variety				
Own tax rate differential	-0.206		-0.216	-0.270
	(0.125)		(0.115)	(0.100)
Other tax rate differential		0.015	0.054	
		(0.106)	(0.093)	
Specification:				
Food dummy	y	y	y	y
Cell (border pair by year) fixed effects				y
N (observations)	8430	8430	8430	8430

Notes: This table reports regressions of prices, quantity, and product variety on average tax rates for food and nonfood products. For each border pair-by-year cell, there are two observations: one for food products and one for nonfood products. All variables are measured as within-cell differences between the two contiguous counties. Own tax rate is the average food tax rate differential for food observations and the average nonfood tax rate differential for nonfood observations. Other tax rate is the average food tax rate differential for nonfood observations and the average nonfood tax rate differential for food observations. Standard errors are clustered at the border pair-by-year cell-level. Each regression includes a dummy variable for food products. Observations are weighted to reflect the number of underlying module-by-store-by-year observations in each cell.

Online Appendix Table OA.2
Variance Decomposition of Tax Rates

Variance of log(1+τ)	0.0010
Standard deviation of log(1+τ)	0.0312
Standard deviation within:	
Store × Year cells	0.0269
Module × Border Pair × Year cells	0.0108
Fraction of variance within:	
Store × Year cells	74.6%
Module × Border Pair × Year cells	11.9%

Notes: This table reports variance decompositions of the tax rate variable in the RMS data.

Online Appendix Table OA.3:
Effect of Sales Taxes on Prices, Quantity, and Product Variety
[Robustness to Dropping Alcohol and Tobacco Product Modules]

Dependent Variable:	Prices	Quantity	Variety
	(1)	(2)	(3)
Panel A: County Border Pair OLS Estimates			
$\log(1+\tau_{mcn})$	0.008	-0.678	-0.261
	(0.011)	(0.137)	(0.060)
Panel B: 2SLS Estimates Using State-Level Tax Rate as Instrument			
$\log(1+\tau_{mcn})$	0.011	-0.736	-0.267
	(0.011)	(0.135)	(0.060)
Specification:			
Store fixed effects	y	y	y
Module × County Border Pair fixed effects	y	y	y

Notes: Sales tax rates are measured annually based on the rates that were effective on September 1. Sales, prices, and variety are measured yearly. The Retail Scanner data is restricted to modules above the 80th percentile of the national distribution of sales. All reported coefficients are simple averages of nine estimated coefficients -- one for each year from 2006 to 2014. The sample is restricted to border counties and observations are weighted by the inverse of number of pairs a store belongs to. Standard errors are clustered two-way at the state-module level and at the border pair by module level. In panel B, the tax rate is instrumented with the state-level, leave-county-out, average tax rate.

Online Appendix Table OA.4: Additional Sensitivity Analysis of Calibration Results

	Baseline calibration	Alternative demand elasticity and tax salience parameters			
Panel A: Calibrated parameters					
Average tax rate, τ_0	0.034	0.034	0.034	0.034	0.034
Tax salience parameter, θ_τ	0.556	**0.500**	**0.612**	**0.556**	**0.556**
Demand elasticity, ϵ_D	1.170	**1.170**	**1.170**	**1.287**	**1.053**
Panel B: Reduced-form estimates					
Pass-through of taxes into pre-tax prices, $d\log(p)/d\log(1+\tau)$	0.039	0.039	0.039	0.039	0.039
Quantity response, $d\log(Q)/d\log(1+\tau)$	-0.731	-0.731	-0.731	-0.731	-0.731
Variety response, $d\log(J)/d\log(1+\tau)$	-0.243	-0.243	-0.243	-0.243	-0.243
Panel C: Model parameters estimated by matching reduced-form estimates					
Markup, $(p - c'(q))/p$	0.080	0.080	0.080	0.080	0.080
Implied conduct parameter, v_q/J	0.092	**0.092**	**0.092**	**0.101**	**0.083**
Inverse elasticity of marginal surplus, ϵ_{ms}	-0.903	**-0.970**	**-0.846**	**-0.903**	**-0.903**
Variety effect parameter, $\tilde{\Lambda}_0$	0.125	**0.366**	**-0.108**	**-0.113**	**0.425**
Panel D: Calibrated welfare formulas					
Full marginal excess burden (MEB) formula, $d\tilde{W}/d\tau$	-0.083	-0.140	-0.028	-0.025	-0.156
Alternative MEB formula benchmarks:					
Harberger/CLK benchmark, $\theta_\tau * \tau_0 * d\log(Q)/d\log(1+\tau)$	-0.014	-0.012	-0.015	-0.014	-0.014
Besley(1989)-style benchmark; i.e., full MEB formula with $\tilde{\Lambda}_0 = 0$	-0.053	-0.051	-0.054	-0.053	-0.053
% difference between full formula and Besley(1989)-style benchmark	57.5%	172.9%	-48.3%	-51.8%	195.3%

Notes: This table reports structural parameter estimates by finding parameters that allow the model to match the reduced-form estimates. The table reports sensitivity to different assumptions on the demand elasticity and the tax salience parameter.

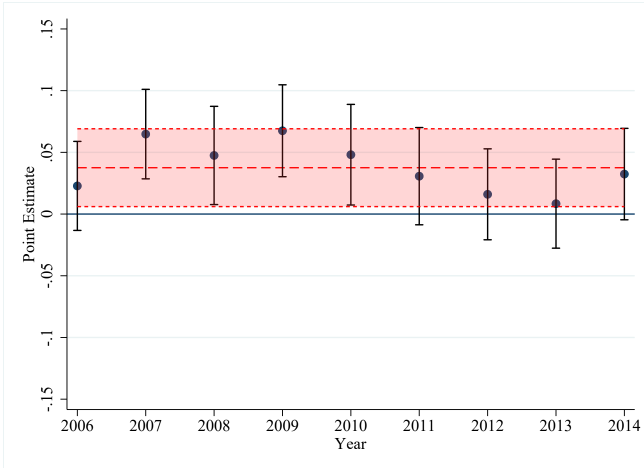
Online Appendix Table OA.5: Counterfactual Scenarios Comparing Ad Valorem and Unit Tax Taxes

Variety effect parameter, $\tilde{\Lambda}_0$	Baseline variety effect estimate, $\tilde{\Lambda}_0 = 0.133$		No variety effect counterfactual, $\tilde{\Lambda}_0 = 0.000$		Large variety effect counterfactual, $\tilde{\Lambda}_0 = 1.000$	
	Ad valorem tax ($d\tau$)	Specific tax (dt)	Ad valorem tax ($d\tau$)	Specific tax (dt)	Ad valorem tax ($d\tau$)	Specific tax (dt)
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Pass-through of taxes into pre-tax prices						
$d\log(p)/d\log(1+\tau)$ or $d\log(p)/dt$	0.039	0.059	0.036	0.060	0.060	0.056
Difference b/w ad valorem and specific tax	-0.020		-0.024		0.004	
Panel B: Marginal cost of public funds (*MCPF*)						
$MCPF_\tau$ or $MCPF_t$	0.082	0.067	0.048	0.072	0.307	0.033
Difference between ad valorem and specific tax	0.015		-0.024		0.273	
Panel C: The effects of taxes on variety and profits						
$d\log(J)/d\log(1+\tau)$ or $d\log(J)/dt$	-0.243	0.037	-0.245	0.037	-0.230	0.035
$\partial\log(\pi)/\partial\log(1+\tau)$ or $\partial\log(\pi)/\partial t$	-0.041	0.006	-0.041	0.006	-0.041	0.006
Panel D: Competitive effects of entry						
$\partial\log(p)/\partial\log(J)$	-0.106	-0.105	-0.092	-0.091	-0.207	-0.204
$\partial\log(q)/\partial\log(J)$	-0.751	-0.740	-0.911	-0.897	0.368	0.363
Stability condition (must be >0)	1.822	1.822	1.806	1.806	1.927	1.927

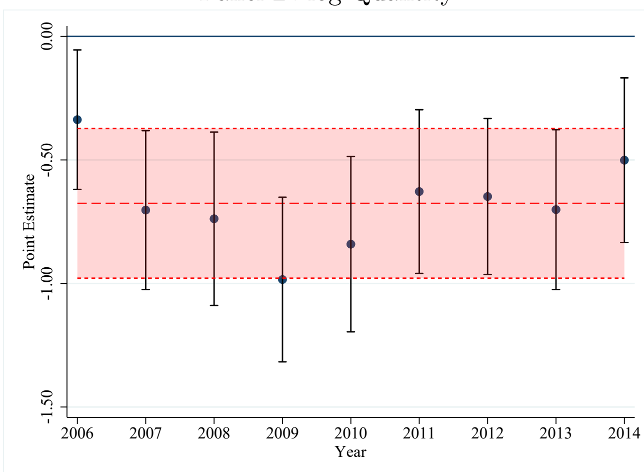
Notes: This table reports counterfactual estimates of reduced-form effects of specific taxes under different assumptions on the variety effect based on using the model parameter estimates of Table 4. The difference between the ad valorem and specific tax *MCPF* estimates ($MCFP_\tau$ - $MCPF_t$) switches sign as the variety effect increases (comparing columns (1) and (2) to (3) and (4)). The difference between ad valorem and specific tax pass-through rate is less sensitive to the variety effect and only switches sign when the variety effect is large (columns (5) and (6)).

Figure OA.1: Year-by-Year OLS Regression Coefficients

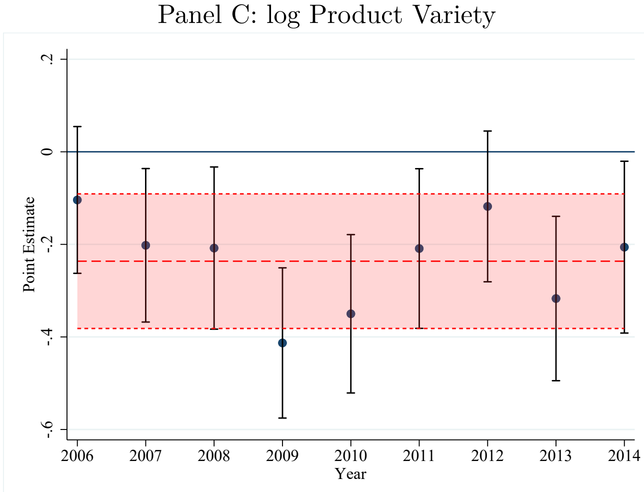
Panel A: log Prices



Panel B: log Quantity



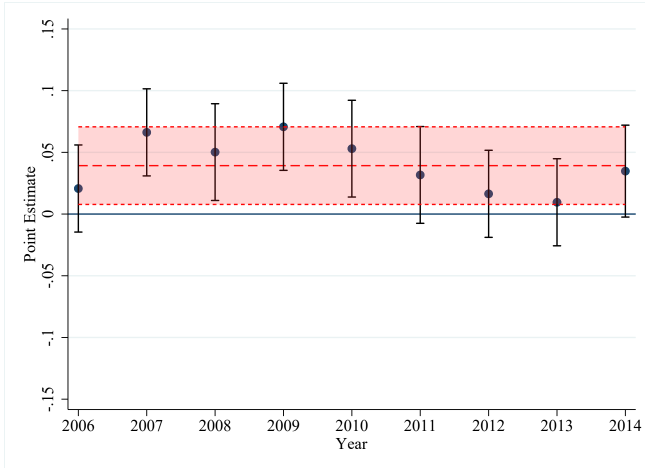
Panel C: log Product Variety



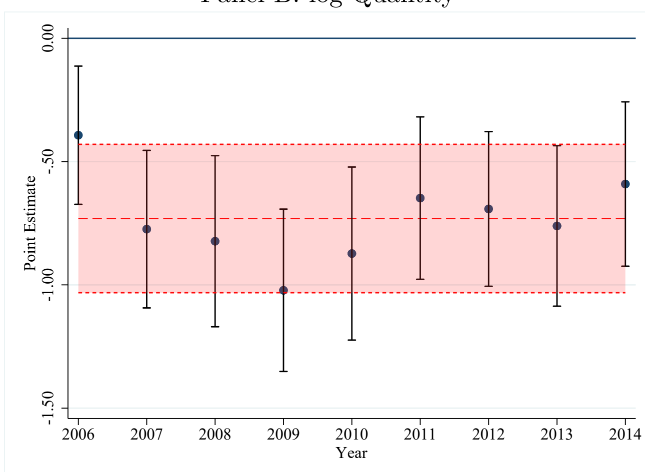
Notes: This figures shows yearly estimates of the effects of sales taxes on price (panel A), quantity (panel B) and product variety (C). All models are based on equation (17) and estimated by OLS. The black vertical bars indicate 95% confidence intervals. The dashed red horizontal line indicates the average coefficient estimate across all 9 years, and the red area denotes the 95% confidence interval around that average.

Figure OA.2: Year-by-Year 2SLS Regression Coefficients

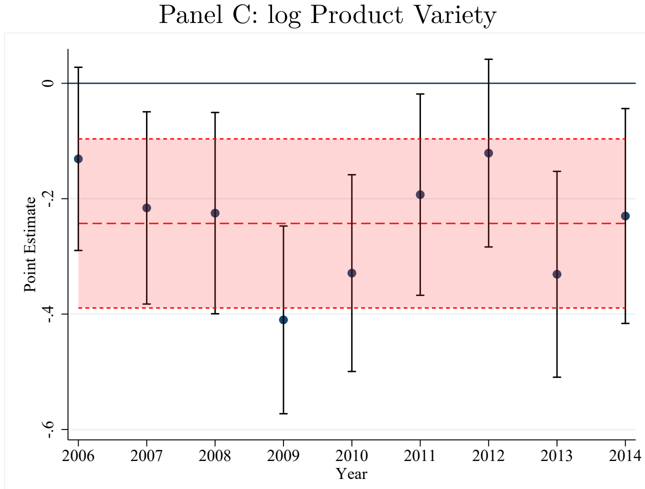
Panel A: log Prices



Panel B: log Quantity



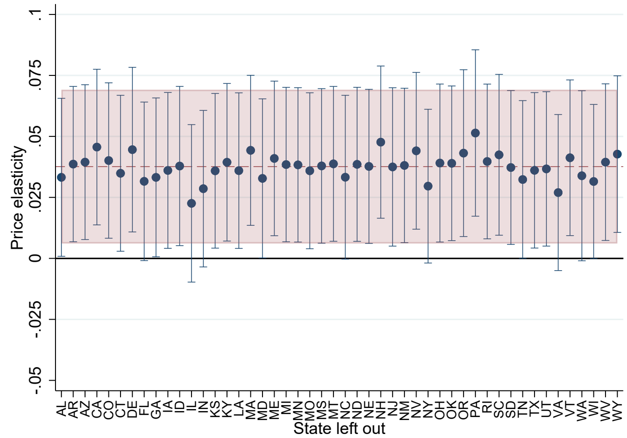
Panel C: log Product Variety



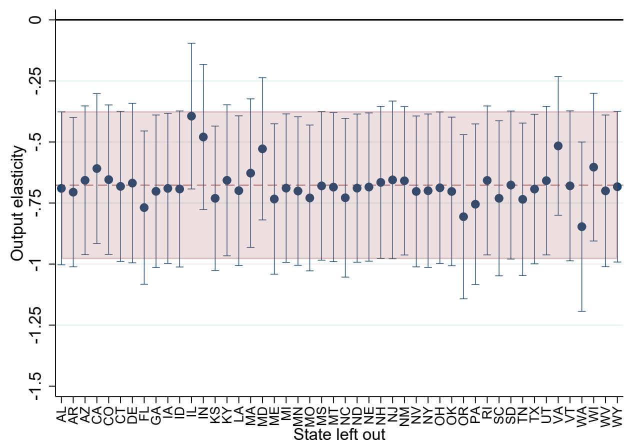
Notes: This figures shows yearly estimates of the effects of sales taxes on price (panel A), quantity (panel B) and product variety (C). All models are based on equation (17) and estimated by 2SLS. The instrument is the average state-level, leave-county-out average tax rate for each module-year cell. The black vertical bars indicate 95% confidence intervals. The dashed red horizontal line indicates the average coefficient estimate across all 9 years, and the red area denotes the 95% confidence interval around that average.

Figure OA.3: Leave-State-Out Regression Coefficients

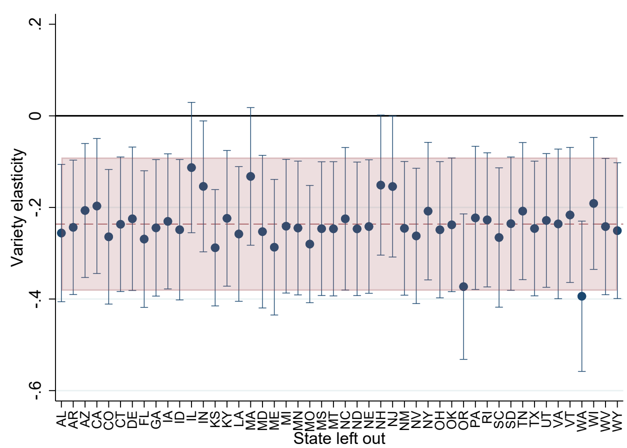
Panel A: log Prices



Panel B: log Quantity

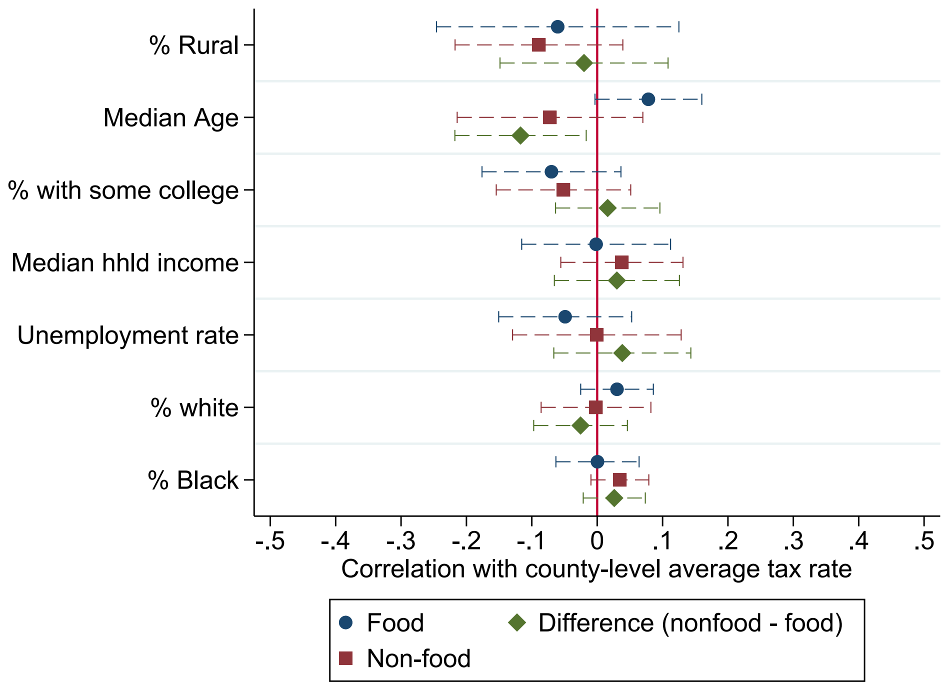


Panel C: log Product Variety



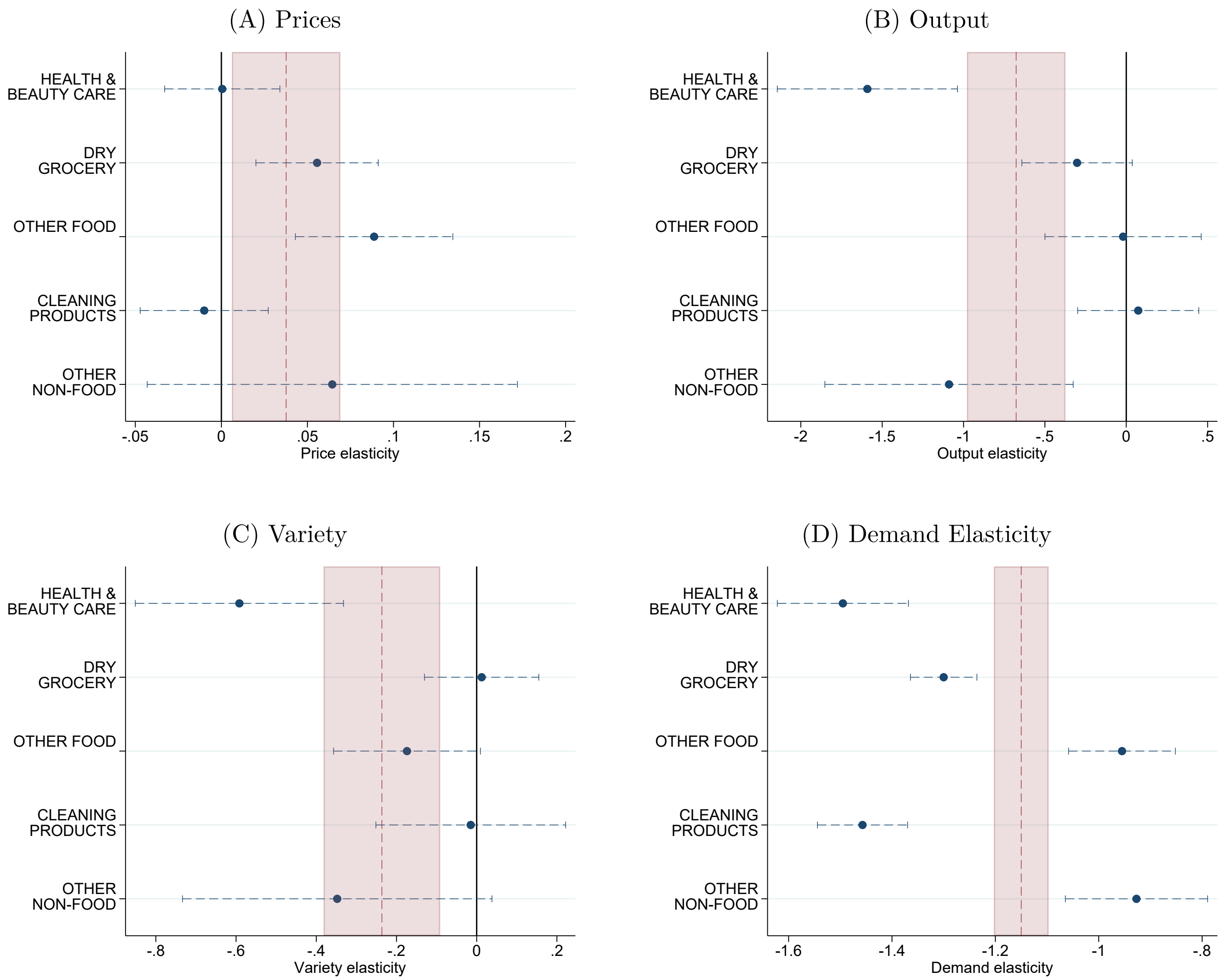
Notes: This figures shows yearly leave-state-out estimates of the effects of sales taxes on price (panel A), quantity (panel B) and product variety (C). All models are based on equation (17) and estimated by OLS. For each regression, all stores located in a given state or in a county adjacent to that state are dropped. The blue vertical bars indicate 95% confidence intervals. The dashed red horizontal line indicates the average coefficient estimate across all 9 years, and the red area denotes the 95% confidence interval around that average.

Figure OA.4: Correlations between County Demographics and Tax Rates



Notes: This figures shows correlation coefficients between county-level demographics (from the American Community Survey) and county-level average sales tax rates in 2008. Blue dots depict correlations with the average tax rate on food products. Red squares depict correlations with the average tax rate on non-food products. Green diamonds depict correlations with the county-specific difference between tax rates on non-food and food products. All correlations are estimated by OLS using a specification that includes border-pair fixed effects. The horizontal dashed bars indicate 95% confidence intervals. Standard errors are clustered at the state level.

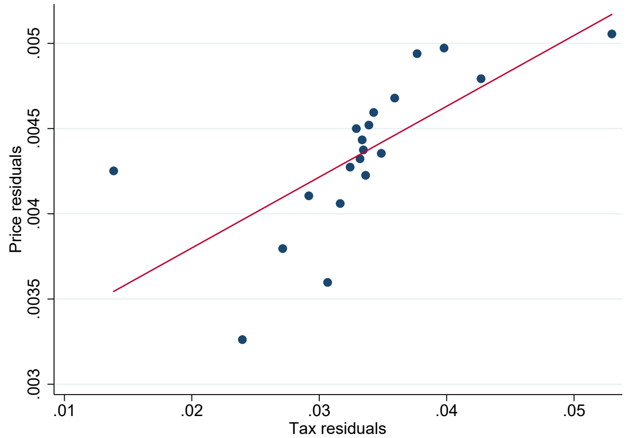
Figure OA.5: Heterogeneity Across Product Categories



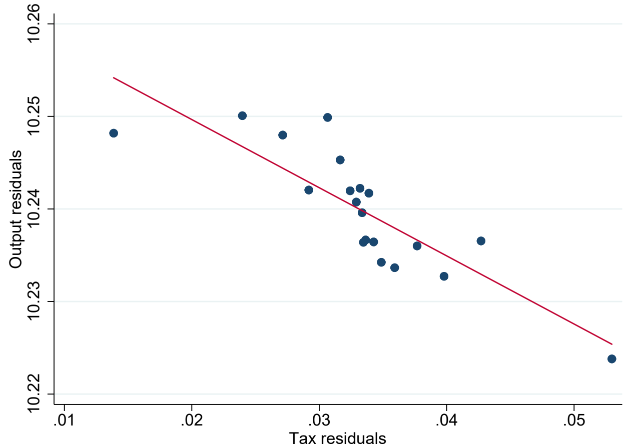
Notes: This figures shows estimates of the effects of sales taxes on price (panel A), quantity (panel B) and product variety (C) for different categories of products. Models for panels A, B and C are based on an augmented version of equation (17), in which tax rates are interacted with indicators for 5 different categories of goods. Panel D shows corresponding estimates of the demand elasticity, estimated using the methods described in Kroft et al. (2021). The blue dashed bars indicate 95% confidence intervals. The red vertical line indicates the average coefficient estimate across all 9 years, and the red area denotes the 95% confidence interval around that average.

Figure OA.6: Binscatter Plots of Regression Residuals

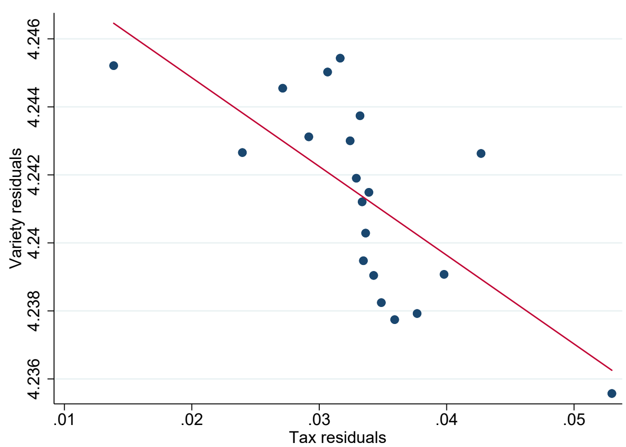
Panel A: log Prices



Panel B: log Quantity



Panel C: log Product Variety



Notes: This figures shows binscatter plots of regression residuals from models estimating the effects of sales taxes on price (panel A), quantity (panel B) and product variety (C). The number of bins is set to 20. All residuals are based on equation (17) and estimated by OLS. The red lines show the linear fit, the slope of which corresponds to our main estimates reported in Table 2.